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COHOMOLOGIES OF HOMOGENEOUS ENDOMORPHISM BUNDLES OVER LOW DIMENSIONAL KÄHLER C-SPACES

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1. Introduction

In this paper, we determine the infinitesimal deformations of an Einstein-Hermitian structure of a homogeneous vector bundle in several cases. In particular, we get the tangent space at the homogeneous structure of the moduli space of Einstein-Hermitian structures as the representation space of a compact Lie group.

A compact simply connected homogeneous Kähler manifold is called a *Kähler C-space*. Such a manifold can be written as G/K where G is a compact semisimple Lie group and K is the centralizer of a toral subgroup of G ([10]). Let G^C be the complexification of G and K^C the complexification of K in G^C. We denote by L the parabolic subgroup of G^C which contains K^C. G/K is diffeomorphic to G^C/L. Thus G/K admits a holomorphic structure from the holomorphic structure of G^C/L. Moreover it admits a G-invariant Kähler metric.

Let (ρ, V) be a complex representation of K. Then (ρ, V) can be extended to a holomorphic representation (ρ_L, V) of L. The homogeneous vector bundle $E_\rho=G\times_\rho V$ is isomorphic to the homogeneous holomorphic vector bundle $E_{\rho_L}=G^C\times_{\rho_L}V$ as C^∞-vector bundles. Thus the homogeneous vector bundle E_ρ has a natural holomorphic structure from the holomorphic structure of E_{ρ_L} ([3]). Moreover if (ρ, V) is irreducible, then E_ρ has a unique G-invariant Einstein-Hermitian structure up to a homothety ([8]).

An irreducible complex representation (ρ, V) is determined by the highest weight. Then a homogeneous vector bundle E_ρ is determined by the highest weight of (ρ, V), if (ρ, V) is irreducible. It is natural to ask how we describe the deformations of the holomorphic structure and the Einstein-Hermitian structure by the highest weight. Also we ask how we describe moduli spaces of holomorphic structures and Einstein-Hermitian structures by the highest weight.

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