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GRADED COALGEBRAS AND MORITA-TAKEUCHI CONTEXTS

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0. Introduction

Viewing a G-graded k-coalgebra over the field k as a right kG-comodule coalgebra it is possible to use a Hopf algebraic approach to the study of coalgebras graded by an arbitrary group that was started in [NT].

Let $C=\bigoplus_{g\in G} C_g$ be a G-graded coalgebra. The graded C-comodules may be viewed as comodules over the smash product $C\rtimes kG$, the general definition of which was given in [M]. Coalgebras graded by an arbitrary group have been considered in [FM] in order to introduce the notion of G-graded Hopf algebras. On the other hand, M. Takeuchi introduced in [T] the sets of pre-equivalence data connecting categories of comodules over two coalgebras (we call such a set a Morita-Takeuchi context). The main result of this note is a coalgebra version of a result established by M. Cohen, S. Montgomery in [CM] for group-graded rings: for a graded coalgebra C the coalgebras C_1 and $C\rtimes kG$ are connected by a Morita-Takeuchi context in which one of the structure maps is injective. Most of the results in this note are consequences of the foregoing. As a first application we find that a coalgebra C is strongly graded if and only if the other structure map of the context is also injective. The final section provides analogues of the Cohen-Montgomery duality theorems: if C is a coalgebra graded by the finite group G of order n, then G acts on the smash coproduct as a group of automorphisms of coalgebras and $(C\rtimes kG)\rtimes kG^*$ is coalgebra isomorphic to the comatrix coalgebra $M^c(n, C)$. If G is a finite group of order n, acting on the coalgebra D as a group of coalgebra automorphisms, then the smash coproduct $D\rtimes kG^*$ is strongly graded by G and moreover: $(D\rtimes kG^*)\rtimes kG\cong M^c(n, D)$. The second duality theorem is again a direct consequence of the Morita-Takeuchi context mentioned above.

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