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SCHWARTZ KERNEL THEOREM FOR THE FOURIER HYPERFUNCTIONS

By

Soon-Yeong CHUNG, Dohan KIM and Eun Gu LEE

§ 0. Introduction

The purpose of this paper is to give a direct proof of the Schwartz kernel theorem for the Fourier hyperfunctions. The Schwartz kernel theorem for the Fourier hyperfunctions means that with every Fourier hyperfunction K in $\mathcal{F}(\boldsymbol{R}^{n_1}\times\boldsymbol{R}^{n_2})$ we can associate a linear map

$$\mathcal{K}:\mathcal{F}(\boldsymbol{R}^{n_2})\longrightarrow\mathcal{F}'(\boldsymbol{R}^{n_1})$$

and vice versa, which is determined by

$$\langle\mathcal{K}\varphi,\ \psi\rangle=K(\psi\otimes\varphi),\qquad \psi\in\mathcal{F}(\boldsymbol{R}^{n_1}),\ \varphi\in\mathcal{F}(\boldsymbol{R}^{n_2}).$$

For the proof we apply the representation of the Fourier hyperfunctions as the initial values of the smooth solutions of the heat equation as in [3] which implies that if a C^∞-solution $U(x, t)$ satisfies some growth condition then we can assign a unique compactly supported Fourier hyperfunction $u(x)$ to $U(x, t)$ (see Theorem 1.4). Also we make use of the following real characterizations of the space $\mathcal{F}$ of test functions for the Fourier hyperfunctions in [1, 3, 5]

$$\mathcal{F}=\left\{\varphi\in C^\infty \,\middle|\, \sup_{\alpha,x}\frac{|\partial^\alpha\varphi(x)|\exp k|x|}{h^{|\alpha|}\alpha!}<\infty \quad \text{for some } h,\ k>0\right\}$$

$$=\{\varphi\in C^\infty \,|\, |\sup|\varphi(x)|\exp k|x|<\infty,\quad \sup|\hat{\varphi}(\xi)|\ \exp h|\xi|<\infty$$

$$\text{for some } h,\ k>0\}$$

Also, we closely follow the direct proof of the Schwartz kernel theorem for the distributions as in Hörmander [2].

§ 1. Preliminaries

We denote by $x=(x_1,\ x_2)\in\boldsymbol{R}^n$ for $x_1\in\boldsymbol{R}^{n_1}$ and $x_2\in\boldsymbol{R}^{n_2}$, and use the multi-index notation; $|\alpha|=\alpha_1+\cdots+\alpha_n$, $\partial^\alpha=\partial^{\alpha_1}\cdots\partial^{\alpha_n}$ for $\alpha=(\alpha_1,\ \cdots,\ \alpha_n)\in\boldsymbol{N}_0^n$ where

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