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ON THE LONG-RANGE SCATTFRING FOR ONE- AND TWO-PARTICLE SCHRÖDINGER OPERATORS WITH CONSTANT MAGNETIC FIELDS*

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1. Introduction

In this paper we prove the asymptotic completeness for the following 1-particle Schrödinger operator with a constant magnetic field $B\in\boldsymbol{R}^3$, $B\neq 0$:

$$H=H_0+V=\frac{1}{2}\left(p-\frac{1}{2}B\times r\right)^2+V(r),$$

where $r\in\boldsymbol{R}^3$ and $p=-i\nabla_r$. The real-valued smooth function $V(r)$ is a long-range potential, that is, we impose the following decay condition on $V(r)$:

(V) *As* $|r|\to\infty$,

(1.1) $$|V(r)|+|r_{\|}\partial_{\|}V(r)|=o(1).$$

Moreover, for some $\delta_0>0$,

(1.2) $$|\partial_{\perp}V(r)|\leqq C\langle r_{\|}\rangle^{-1-\delta_0},$$

(1.3) $$|\partial_{\|}^{l}V(r)|\leqq C_l\langle r_{\|}\rangle^{-l-\delta_0} \quad \textit{for any integer } l\geqq 0.$$

Here $r_{\|}=r\cdot B/|B|$, and $r_{\perp}$ denotes the component of r perpendicular to B; $\partial_{\perp}$ and $\partial_{\|}$ denote the partial differentials with respect to the variables $r_{\perp}$ and $r_{\|}$, respectively. We use $\langle r\rangle$ for $(1+|r|^2)^{1/2}$ throughout this paper. As will be easily seen from below, $V(r)$ is allowed to include short-range parts with some local singularities.

In the absence of magnetic fields, scattering theory is quite well understood for 2-body Schrödinger operators with a large class of long-range potentials and, recently, the long-range scattering for constant electric fields have been intensively investigated by several authors (cf. [5, 8, 9, 16]). On the other hand, the asymptotic completeness for long-range Schrödinger operators with constant magnetic fields was first proved by Avron-Herbst-Simon in [1]; they

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