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ON THE GAUSS MAP OF SURFACES OF REVOLUTION IN A 3-DIMENSIONAL MINKOWSKI SPACE

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§ 1. Introduction.

For the Gauss map of a surface of revolution in $\boldsymbol{R}^3$ the following theorem is proved by Dillen, Pas and Verstraelen [3].

THEOREM A. *The only surfaces of revolution in* $\boldsymbol{R}^3$ *whose Gauss map* ξ *satisfies*

$$(1.1) \qquad \Delta\xi = A\xi, \qquad A \in Mat(3, \boldsymbol{R})$$

are locally the plane, the sphere and the circular cylinder.

In the case of a Minkowski space, a Gauss map is defined as follows. Let $\boldsymbol{R}_1^{n+1}$ be an $(n+1)$-dimensional Minkowski space with standard coordinate system $\{x_A\}$ whose line element ds^2 is given by $ds^2 = -(dx_0)^2 + \sum_{i=1}^{n}(dx_i)^2$. Let $S_1^n(c)$ (resp. $H^n(c)$) be an n-dimensional de Sitter space (resp. a hyperbolic space) of constant curvature c in $\boldsymbol{R}_1^{n+1}$. We denote by $M^n(\varepsilon)$ a de Sitter space $S_1^n(1)$ or a hyperbolic space $H^n(-1)$, according as $\varepsilon=1$ or -1. Let M be a n-dimensional space-like or time-like hypersurface in $\boldsymbol{R}_1^{n+1}$ and ξ a unit vector field normal to M. Then, for any point p in M, we can regard $\xi(p)$ as a point in $H^n(-1)$ or $S_1^n(1)$ by translating parallelly to the origin in the ambient space $\boldsymbol{R}_1^{n+1}$, according as the surface M is space-like or time-like. The map ξ of M into $M^n(\varepsilon)$ is called a *Gauss map* of M into $\boldsymbol{R}_1^{n+1}$.

As a Lorentz version of Baikoussis and Blair's result [1], the author [2] proves the following

THEOREM B. *The only space-like or time-like ruled surfaces in* $\boldsymbol{R}_1^3$ *whose Gauss map* $\xi: M \to M^2(\varepsilon)$ *satisfies* (1.1) *are locally the following spaces*:

i. $\boldsymbol{R}_1^2$, $S_1^1 \times \boldsymbol{R}^1$ *and* $\boldsymbol{R}_1^1 \times S^1$ *if* $\varepsilon=1$,

ii. $\boldsymbol{R}^2$ *and* $H^1 \times \boldsymbol{R}^1$ *if* $\varepsilon=-1$.

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