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# REMARKS ON $d$-GONAL CURVES

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## § 0. Introduction.

Let $M$ be a compact Riemann surface and $f$ be a meromorphic function on $M$. We denote the principal divisor associated to $f$ by $(f)$ and the polar divisor of $f$ by $(f)_\infty$. If $d$=degree of the divisor $(f)_\infty$, we call $f$ a meromorphic function of degree $d$. If $d$ is the minimal integer in which a non-trivial meromorphic function $f$ of degree $d$ exists on $M$, then we call $M$ a $d$-gonal curve. In this case the complete linear system $|(f)_\infty|$ has projective dimension one. Moreover if $f$ defines a cyclic covering $M\to \boldsymbol{P}_1$ over a Riemann sphere $\boldsymbol{P}_1$, then we call $M$ a cyclic $d$-gonal curve.

Now we assume that $M$ is a $p$-gonal curve of genus $g$ with a prime number $p$. Then Namba has shown that $M$ has a unique linear system $g_p^1$ of projective dimension one and degree $p$ provided $g>(p-1)^2$ ([6]). For example if $M$ is defined by an equation $y^p-(x-a_1)^{r_1}\cdots(x-a_s)^{r_s}=0$ with $(p, r_i)=1$, $\Sigma r_i\equiv 0 \pmod p$ and $s\geqq 2p+1$, then $M$ is $p$-gonal and having a unique $g_p^1$ ([7]).

In this paper we treat a compact Riemann surface $M$ defined by an equation;

$$y^d-(x-a_1)^{r_1}\cdots(x-a_s)^{r_s}=0 \qquad *)$$
$$\text{with} \quad \Sigma r_i\equiv 0 \bmod d \quad \text{and} \quad 1\leqq r_i<d,$$

where $d$ is not necessarily a prime number.

In § 2, we will show that $M$ is $d$-gonal with the function $x$ of degree $d$ if there are enough $r_i$'s relatively prime to $p$ for each prime number $p$ dividing $d$. In this case we call $M$ a cyclic $d$-gonal curve. We will also show that $M$ has a unique $g_d^1$ if there are more sufficient such $r_i$'s as above (§ 2).

In § 3, let $M$ be a cyclic $d$-gonal curve defined by $*)$ having a unique $g_d^1$ and $M'$ be a compact Riemann surface defined by $y^d-(x-b_1)^{t_1}\cdots(x-b_s)^{t_s}=0$. We will study the relations among $a_i$, $b_i$, $r_i$ and $t_i$ $(1\leqq i\leqq s)$ in the case $M$ and $M'$ are conformaly equivalent. Namba [7] and Kato [5] have already studied this problem in the case $d$ is a prime number. We will give similar results for an arbitrary $d$ (§ 3).

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