TSUKUBA J. MATH.
Vol. 19 No. 2 (1995), 285–304

ON THE GAUSS MAP OF RULED SURFACES IN A 3-DIMENSIONAL MINKOWSKI SPACE

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§1. Introduction.

Relative to Takahashi's theorem [9] for minimal submanifolds, the idea of submanifolds of finite type in a Euclidean space was introduced by Chen [2] and the theory is recently greatly developed. Let $x: M \to \boldsymbol{R}^{n+1}$ be an isometric immersion of n-dimensional Riemannian manifold into an $(n+1)$-dimensional Euclidean space $\boldsymbol{R}^{n+1}$ and Δ the Laplacian on M. As a generalization of Takahashi's theorem for the case of hypersurfaces, Garay [4] considered the hypersurface satisfying the condition $\Delta x = Ax$, where A denotes the constant diagonal matrix of order $n+1$.

On the other hand, let $x: M \to \boldsymbol{R}^m$ be an isometric immersion of a compact oriented n-dimensional Riemannian manifold into $\boldsymbol{R}^m$. For a generalized Gauss map $G: M \to G(n, m) \subset \boldsymbol{R}^N$ $\left(N = \binom{m}{n}\right)$ of x, where $G(n, m)$ is the Grassmann manifold consisting of all oriented n-planes through the origin of $\boldsymbol{R}^m$, Chen and Piccinni [3] characterized the submanifold satisfying the condition $\Delta G = \lambda G$ $(\lambda \in \boldsymbol{R})$. For a hypersurface M in $\boldsymbol{R}^{n+1}$ and a unit vector field ξ normal to M, we can regard $\xi(p)$ $(p \in M)$ as a point in an n-dimensional unit sphere $S^n(1)$ by translating parallelly to the origin in the ambient space $\boldsymbol{R}^{n+1}$. The map ξ of M into $S^n(1)$ is called a *Gauss map* of M in $\boldsymbol{R}^{n+1}$. Recently for the Gauss map of a surface in $\boldsymbol{R}^3$ the following theorem is proved by Baikoussis and Blair [1].

THEOREM. *The only ruled surfaces in $\boldsymbol{R}^3$ whose Gauss map ξ satisfies*

$$\Delta\xi = A\xi, \qquad A \in Mat\ (3, \boldsymbol{R}) \tag{1.1}$$

are locally the plane and the circular cylinder.

It seems to be interesting to investigate the Lorentz version of the above theorem. Now, let $\boldsymbol{R}_1^{m+1}$ be an $(m+1)$-dimensional Minkowski space with standard coordinate system $\{x_A\}$ whose line element ds^2 is given by $ds^2 = -(dx_0)^2 +$

Partially supported by TERC-KOSEF.
Received August 20, 1993.