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ON THE QUATERNIONIC SECTIONAL CURVATURE OF AN INDEFINITE QUATERNIONIC KÄHLER MANIFOLD

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1. Introduction

The quaternionic sectional curvature of an indefinite quaternionic Kähler manifold is investigated in [6], where it is shown that its treatment presents certain analogies with that of the holomorphic sectional curvature of an indefinite Kähler manifold [1].

An important feature of indefinite metrics is the existence of null geodesics, and the study of the Jacobi operator along such geodesics. A simple examination of the curvature tensor of an indefinite Kähler manifold of constant holomorphic sectional curvature shows that its restriction to degenerate holomorphic planes vanishes identically. Such condition $R(U, JU, JU, U)=0$ is shown in [3] to be strictly weaker than constant holomorphic sectional curvature. In fact, the product $M_1(c)\times M_2(c)$ of two positive definite Kähler manifolds endowed with the metric $g=g_1\oplus(-g_2)$ satisfies $R(U, JU, JU, U)=0$ but its holomorphic sectional curvature is not constant, unless $c=0$.

When one considers an indefinite quaternionic Kähler manifold of constant q-sectional curvature, the curvature tensor is expressed in terms of the metric and the almost complex structures of the quaternionic structure. From that expression it immediately follows that

$$R(U, \phi U, \phi U, U)=0, \qquad \phi=I, J, K, \tag{1}$$

where $\{I, J, K\}$ is any local basis of the bundle of almost complex structures on M.

The aim of this paper is to investigate such condition (1), and to prove that it is characteristic of indefinite quaternionic space forms. This makes a significant difference in the study of the curvature of indefinite quaternionic Kähler manifolds with respect to the complex case. We will show the following.

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