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ON SOME CLASSES OF ALMOST CONTACT METRIC MANIFOLDS

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1. Introduction

In [1] J. Berndt and L. Vanhecke introduced two classes ($\mathfrak{C}$- and $\mathfrak{B}$-spaces) of Riemannian manifolds which include the class of locally symmetric spaces using the properties of Jaoobi operators along geodesics. They provided some characterizations of $\mathfrak{C}$- and $\mathfrak{B}$-spaces and gave the classifications for dimensions two and three. For further developments on the two spaces, we refer to [2], [3] and [8]. Further, T. Takahashi ([19]) introduced the notion of a (Sasakian) locally φ-symmetric space which may be considered as the analogue in the almost contact metric case of locally Hermitian symmetric spaces. Also he gave examples and equivalent properties of Sasakian locally φ-symmetric spaces. For further results about the Sasakian locally φ-symmetric spaces, we refer to [5], [6].

In the present paper, we introduce in an analogous way as in [1] four classes of almost contact metric manifolds involving Sasakian locally φ-symmetric spaces. In section 2, we recall definitions and several elementary properties of an almost contact, a contact, a K-contact metric manifold and a Sasakian manifold. In sections 3 and 4 we give the definitions of a $\mathfrak{D}\mathfrak{C}$-*space*, a $\mathfrak{D}\mathfrak{P}$-*space*, a $\xi\mathfrak{C}$-*space* and a $\xi\mathfrak{P}$-*space* which are almost contact metric analogues of a $\mathfrak{C}$-space or a $\mathfrak{P}$-space in the Riemannian case. We may observe that a Sasakian manifold is a $\xi\mathfrak{C}$-space and at the same time a $\xi\mathfrak{P}$-space. Also we prove that a Sasakian manifold is locally φ-symmetric if and only if it is a $\mathfrak{D}\mathfrak{C}$-space and at the same time a $\mathfrak{D}\mathfrak{P}$-space. In section 5, we show that the tangent sphere bundle of a 2-dimensional Riemannian manifold is a $\xi\mathfrak{P}$-space if and only if the base manifold is flat or of constant curvature 1. Furthermore, we give some examples of almost contact metric $\mathfrak{D}\mathfrak{C}$-spaces and $\mathfrak{D}\mathfrak{P}$-spaces. In section 6, we consider real hypersurfaces of a complex projective space CP^n with Fubini-Study metric and determine $\xi\mathfrak{P}$-hypersurfaces of CP^n. We also show that a homogeneous real hypersurface of CP^n is a $\xi\mathfrak{C}$-space, and moreover, we give

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