A NORMALIZATION-PROCEDURE FOR THE FIRST ORDER CLASSICAL NATURAL DEDUCTION WITH FULL LOGICAL SYMBOLS

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§ 0. Introduction

The system of natural deduction was introduced by Gentzen [1]. He also introduced the system of sequent calculus in order to prove his *Hauptsatz* which states every proof can be reduced to a proof without roundabouts. (In some cases, the *Hauptsatz* is called the *cut-elimination theorem* or the *normalization theorem*.) His system of natural deduction was not suitable for the *Hauptsatz* in the case of classical logic, because in the system the classical logic was formalized as the intuitionistic logic with the law of the excluded middle. Prawitz [2], [3] settled this trouble in his system of natural deduction by formalizing the classical logic as the minimal logic with classical absurdity rule. However his solution was a partial one, since his system of classical logic did not have the logical symbols for the disjunction and for the existential quantifier as the primitive logical symbols. Seldin [4], [5] and Stålmarck [6] proved the normalization theorem for the first order classical natural deduction with full logical symbols. But the reduction procedures defined by them are complicated in comparison with Prawitz's one.

In this paper, we define another reduction procedure for the first order classical natural deduction with full logical symbols. It is as simple as Prawitz's one is. In other words, our reduction procedure is a natural extension of the Prawitz's. Our proof of the normalization theorem will be done simultaneously for the intuitionistic logic and for the classical logic, as the Gentzen's proof of the *Hauptsatz* was. Notice that our normalization theorem is one of the so called *weak normalization* theorems.

§ 1. System

The system used in this paper is the first order classical logic formalized in the style of natural deduction. It has all logical symbols as primitive ones.