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ON THE LAPLACIAN ON A SPACE OF WHITE NOISE FUNCTIONALS

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§ 1. Introduction.

We are greatly interested in the Laplacian on a space of white noise functionals. To have in mind aspects of application to mathematical physics, we can say that it is common in general to use the weak derivative D on a given basic Hilbert space, so as to define d_p which just corresponds to the de Rham exterior differential operator. In doing so, one of the remarkable characteristics of our work consists in adoption of the Hida differential ∂_t instead of D. This distinction from other related works does provide a framework of analysis equipped with the function for perception of the time t, with the result that it is converted into a more flexible and charming theory which enables us to treat time evolution directly. It can be said, therefore, that our work is successful in deepening works about the general theory done by Arai-Mitoma [2], not only on a qualitative basis but also from the applicatory point of view in direct description of operators in terms of time evolution.

The differential ∂_t has its adjoint operator ∂_t^* in Hida sense and it is called the Kubo operator. Indeed, ∂_t^* is realized by extending the functional space even into the widest one $(E)^*$, where a Gelfand triple $(E) \subsetneq (L^2) \subsetneq (E)^*$ is a fundamental setting in white noise analysis, in accordance with our more general choice of the basic Hilbert space H. On the contrary, we define the adjoint operator $d_p{}^*$ of d_p associated with ∂_t without extending the space up to that much. Consequently the Laplacian Δ_p constructed in such an associated manner with d_p (so that, with ∂_t) is realized as an operator having analytically nice properties, such as C^∞-invariance, etc. On the other hand, when we take the Kubo operator as its adjoint, then the so-called Hida Laplacian Δ_H is naturally derived. It is, however, well-known that Δ_H is an operator which maps (S) into $(S)^*$, or in our general setting from (E) into $(E)^*$, which means that it

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