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STABILITY OF MINIMAL SUBMANIFOLDS IN SYMMETRIC SPACES

By

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1. Introduction.

We determine the stability of totally geodesic submanifolds in a compact symmetric space, which are called polars and meridians (see 2.1). These subspaces were introduced by Chen and Nagano ([CN-1]) and we have many interesting results after that ([CN-2], [N-1], [N-2], [NS-1], [NS-2] and [NS-3]). Recently, several results have been obtained about the stability of totally geodesic submanifolds in compact symmetric spaces. Ohnita gave the formula for the index, the nullity and the Killing nullity of a totally geodesic submanifold in a compact symmetric space in [O], in which he also proved that the Helgason sphere in a compact symmetric space is stable. Tasaki proved that the Helgason sphere in a compact Lie group is homologically volume-minimizing in its real homology class in [Ts-1]. He used the calibration theory. And there are studies about the stability of certain closed subgroups in a compact Lie group by Mashimo and Tasaki ([MT-1] and [MT-2]). Mashimo determined all the unstable Cartan embeddings of compact symmetric spaces in [M]. And there is a result about the stability of symmetric R-spaces in Hermitian symmetric spaces and totally complex submanifolds in quaternionic Kähler symmetric spaces of classical type by Takeuchi ([Tk-2]). Recently Nagano and the author have obtained a result on a relationship between the stability of minimal submanifolds and that of p-harmonic maps ([NS-3]). In the present paper we study the stability of all the polars and meridians in every compact symmetric space by using Ohnita's method in Section 3. We will also study the stability of totally complex totally geodesic submanifolds in quaternionic Kähler symmetric spaces of exceptional type in Section 4.

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