TSUKUBA J. MATH.
Vol. 19 No. 1 (1995), 1—13

GORENSTEIN BALANCE OF HOM AND TENSOR

By

Edgar E. ENOCHS and Overtoun M. G. JENDA

By Auslander and Bridger ([1]), Proposition 3.8c)), a finitely generated left R-module C has Gorenstein dimension 0 if and only if there is an exact sequence

$$\cdots \longrightarrow P^{-2} \longrightarrow P^{-1} \longrightarrow P^{0} \longrightarrow P^{1} \longrightarrow \cdots$$

of finitely generated projective left R-modules such that $C=\ker(P^0 \to P^1)$ and such that the dual sequence

$$\cdots \longrightarrow (P^1)^* \longrightarrow (P^0)^* \longrightarrow (P^{-1})^* \longrightarrow \cdots$$

is also exact.

In attempting to dualize the notion of Gorenstein dimension we called such modules C Gorenstein projective modules (see [8]) and then defined Gorenstein injective modules.

Auslander and Buchweitz showed that over certain rings all finitely generated modules have Gorenstein projective precovers (over Cohen-Macauley rings these are their maximal Cohen-Macauley approximations).

An application of their argument shows that over an n-Gorenstein ring all modules have Gorenstein injective preenvelopes.

Over a ring where these precovers and preenvelopes exist, we can apply methods of relative homological algebra (see Eilenberg and Moore [5]) and compute derived functors.

We can then raise the question of balance in the sense of Enochs and Jenda [6]. We can now show that $\mathrm{Hom}(-,-)$ is right balanced by Gorenstein projective and injective modules on a suitable category. Similarly we show that $-\otimes-$ is left balanced by finitely generated Gorenstein projective modules (left and right).

1. Gorenstein Injective and Projective Resolutions.

In the following, module will mean left R-module for some ring R (unless otherwise specified).

Received February 3, 1993. Revised September 20, 1993.