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THE LIMITING AMPLITUDE PRINCIPLE FOR THE ACOUSTIC WAVE OPERATORS IN TWO UNBOUNDED MEDIA

By

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1. Introduction.

In our previous paper Kadowaki [3], we have proved the nonexistence of eigenvalues and the limiting absorption principle for the acoustic wave operators in two unbounded media. In the present paper we study the limiting amplitude principle for these operators. We assume that the propagation speed is discontinuous at the interface and the equilibrium density is 1.

Let $n\geqq 3$ and $x=(y, z)\in \boldsymbol{R}^{n-1}\times\boldsymbol{R}$. We deal with the asymptotic behaviour (as $t\rightarrow+\infty$) of the solutions of the following Cauchy problem

$$(1.1)\quad \begin{cases} \partial_t^2 u(t, x)-a(x)^2\Delta u(t, x)=\exp(-it\sqrt{\omega})f(x) & (t, x)\in \boldsymbol{R}_+\times\boldsymbol{R}^n, \\ u(0, x)=\partial_t u(0, x)=0, \end{cases}$$

where $\omega>0$.

We make the assumptions for the interface separating two media and $a(x)$.

Let $\varphi_0(y)=a|y|$ and $\varphi(y)\in C^1(\boldsymbol{R}^{n-1}\backslash 0)$, where $a\geqq 0$. We assume that $\varphi(y)$ describes the interface and satisfies

$$(\text{A.0})\qquad \sum_{|\alpha|\leq 1}|y|^{|\alpha|}|\partial^\alpha(\varphi(y)-\varphi_0(y))|=O(|y|^{-\theta})\qquad (|y|\rightarrow\infty),$$

for some $\theta>0$, and

$$(\text{A.1})\qquad \sum_{|\alpha|\leq 1}|y|^{|\alpha|}|\partial^\alpha\varphi(y)|=O(|y|^{-\sigma})\qquad (|y|\rightarrow 0),$$

where $0<\sigma<1/2$. For $\varphi(y)$, we use the following notation

$$\Omega_+=\{x=(y, z): z>\varphi(y)\},$$

$$\Omega_-=\{x=(y, z): z<\varphi(y)\},$$

$$S=\{x=(y, z): z=\varphi(y)\}.$$

We denote the unit normal vector at the point $x\in S$ by $\nu=(\nu_1, \nu_2, \cdots, \nu_{n-1}, \nu_z)$

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