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MINIMAL MODELS OF MINIMAL THEORIES

By

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1. Introduction

The algebraic closure $\bar{\mathbf{Q}}$ of the rationals $\mathbf{Q}$ in the complex number field $\mathbf{C}$ is small in the following two senses: (i) There is no proper elementary subfield K of $\bar{\mathbf{Q}}$, and (ii) every field which is elementarily equivalent to $\bar{\mathbf{Q}}$ has a copy of $\bar{\mathbf{Q}}$ in it. In general model theory we have to distinguish these two notions. The notion expressing the first property is called *minimal*, and the other for the the second *prime* (see Definition 1). The following is an example of a theory having a minimal non-prime model:

EXAMPLE (Fuhrken [2]). The theory T_0 is defined as follows: For each $\nu\in{}^{<\omega}2$ we define a function $F_\nu: {}^{\omega}2\to{}^{\omega}2$ by $(F_\nu(\eta))(i)=\nu(i)+\eta(i) \bmod 2$ for $\eta\in{}^{\omega}2$, $i<\omega$. And for $\eta\in{}^{<\omega}2$, $P_\eta=\{\tau\in{}^{\omega}2: \eta\prec\tau\}$. Let $M=({}^{\omega}2, \{F_\nu\}_{\nu\in{}^{<\omega}2}, \{P_\eta\}_{\eta\in{}^{<\omega}2})$ and $T_0=Th(M)$. Then each model generated by only one element ($\in M$) is minimal and non-prime.

Our concern is the number of minimal models of a theory with no prime model (In fact if a theory has a prime model then it has at most one minimal model). In [3] Marcus showed that if T is a theory of one unary function symbol and T has a minimal non-prime model then T has $2^{\aleph_0}$ such models. On the other hand, Shelah proved that for every κ, $1\leqq\kappa\leqq\aleph_0$, there is a theory with exactly κ minimal non-prime models (see [4]).

Here we extent Marcus' result: Theories of one unary function symbol may have the Lascar rank greater than 1 ($U(T)>1$), however if such a theory T has a minimal model then any element a of the model has the minimum Lascar rank (i.e. $U(a)\leqq 1$). Moreover a theory of one unary function symbol is *trivial* (*see* Definition 3). In this paper we show that if a trivial theory T has a minimal non-prime model and every element of the model has the minimum Lascar rank then T has $2^{\aleph_0}$ minimal models. Our result does not depend on the language.

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