

GRADED COALGEBRAS

By

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1. Introduction

The notion of graded coalgebra does not appear very frequently in the literature of coalgebras, neither in papers nor in books (e. g. the well known references [1], [10]). In the mentioned books one can find considered the case when the group is \mathcal{Z} and the components of negative degree are zero. The aim of this paper is to investigate the general concept of graded coalgebra over arbitrary groups and expound their more important properties. A very remarkable point of this paper is the study of the so-called “strongly graded coalgebras” (see §5). The principal ideas, that we use to obtain the main results of this paper, come from the theory of graded rings (see [7]); these become useful by a clever interpretation of the codual methods. We finally remark that a graded coalgebra is a comodule coalgebra over the Hopf algebra $k[G]$, but we will not apply this idea here.

After the introduction and a section where we fix the notation and preliminaries, we give the general properties of graded coalgebras. Proposition 3.1 is the main tool in the computations of the rest of the paper. We show that the functor $U: gr^C \rightarrow M^C$ admits a right adjoint functor F ; moreover if the group G is finite, then F is also a left adjoint functor.

In Section 4, using the cotensor product introduced by Takeuchi in [11] we define the induced functor and we study the more important properties of this functor.

In Section 5 we consider the strongly graded coalgebras. We obtain some nice results about the relation between the categories M^{C_1} and gr^C .

In the next section, we associate a graded ring R to any graded coalgebra C and we study the connexion between the categories $R-gr$ and gr^C . We conclude the paper with examples of graded coalgebras, some of them are very well-known.

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