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ON THE INTERVALS BETEEN CONSECUTIVE NUMBERS THAT ARE SUMS OF TWO PRIMES

By

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1. Introduction.

It is the well known conjecture of H. Cramér that

$$p_{n+1}-p_{n}\ll(\log p_{n})^{2}$$

where p_{n} is the n-th prime. In 1940 P. Erdös proposed the problem to estimate the sum

$$\sum_{p_{n}\leq x}(p_{n+1}-p_{n})^{2},$$

and A. Selberg showed that it is

$$\ll x(\log x)^{3}$$

under the Riemann hypothesis. This problem has been stimulating the several authors, vide [2, 3, 10, 11, 13].

Let (g_{n}) denote in ascending order even integers that are representable as the sum of two primes. The Goldbach conjecture is then interpreted as that

$$g_{n+1}-g_{n}=2$$

for all n. In 1952 Ju. V. Linnik [7] proved, on assuming the Rieman hypothesis, that

$$g_{n+1}-g_{n}\ll(\log g_{n})^{3+\varepsilon}$$

for any $\varepsilon>0$ and all n. Also see [1]. In this paper we shall estimate the third moment of it.

THEOREM.
$$\sum_{g_{n}\leq x}(g_{n+1}-g_{n})^{3}\ll x(\log x)^{300}.$$

COROLLARY. *For* $0\leqq\gamma<3$, *we have*

$$\sum_{g_{n}\leq x}(g_{n+1}-g_{n})^{\gamma}=(2^{\gamma-1}+o(1))x.$$

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