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# ON DILATION THEOREMS OF OPERATOR ALGEBRAS

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## 1. Introduction.

Let $\mathcal{H}$ be a separable, complex Hilbert space and let $\mathcal{L}(\mathcal{H})$ be the algebra of all bounded linear operators on $\mathcal{H}$. For a linear manifold $\mathcal{A}$ in $\mathcal{L}(\mathcal{H})$, a *form* on $\mathcal{A}$ is a linear functional on $\mathcal{A}$. For $x, y \in \mathcal{H}$, $x \otimes y$ denotes the form on $\mathcal{L}(\mathcal{H})$ defined by $x \otimes y(S) = (Sx, y)$, for any $S \in \mathcal{L}(\mathcal{H})$ (cf. [2]). An *elementary form* on a linear manifold $\mathcal{A}$ in $\mathcal{L}(\mathcal{H})$ is the restriction $x \otimes y | \mathcal{A}$ to $\mathcal{A}$ for $x, y \in \mathcal{H}$ (cf. [13]). It is well-known that there are several Hausdroff locally convex topologies on $\mathcal{L}(\mathcal{H})$ (cf. [9]). In particular, a *dual algebra* is a subalgebra of $\mathcal{L}(\mathcal{H})$ that contains the identity operator $I_{\mathcal{H}}$ and is closed in the weak*-topology on $\mathcal{L}(\mathcal{H})$. The theory of dual algebra is closely related to the study of simultaneous equations of weak*-continuous elementary forms (cf. [1], [3], [7], and [10]). Recently several functional analysists have been studied systems of simultaneous equations of weak*-continuous elementary forms on a singly generated dual algebra (cf. [5]). This study has been applied to invariant subspaces, dilation theory, and reflexivity for contraction operators (cf. [5]). In particular, Bercovici-Foias-Pearcy (cf. [4]) obtained several dilation theorems of contraction operators. As a sequel study, Jung-Jo (cf. [12]) studied universal dilation theorems of a contraction operator with some properties. Moreover, M. Marsalli (cf. [13]) studied the dilation theory of general dual algebras with applications to the reflexivity.

This paper is a sequel study of those in [13]. In section 2, we introduce properties $(\tau_{m,n})$ which are concerned with the system of simultaneous equations of vector forms and obtain some related fundamental structure theorems. In section 3, we obtain some new dilation theorems of operator algebras with properties $(\tau_{m,n})$, which are applied to singly generated dual algebras. In section 4, we characterize properties $(\tau_{1,n})$ to dilations of operator algebras. Finally, in section 5, using these results, we obtain a dilation theorem of a contraction operator in the class $A_{1,n}$ which will be defined below and appeared frequently in the theory of dual algebras.

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