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THE LIMITING ABSORPTION PRINCIPLE FOR THE ACOUSTIC WAVE OPERATORS IN TWO UNBOUNDED MEDIA

Dedicated to Professor Mutsuhide Matsumura on his sixtieth birthday

By

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1. Introduction.

In the present paper we study the limiting absorption principle for the acoustic wave operators in two unbounded media. We assume that the propagation speed is discontinuous at the interface and the equilibrium density is 1.

Let $n\geqq 2$ and $x=(y, z)\in \boldsymbol{R}^{n-1}\times\boldsymbol{R}$. The following equation describes the wave propagation here:

$$\partial_t^2 u(t, x)-a(x)^2\Delta u(t, x)=0, \qquad (t, x)\in\boldsymbol{R}\times\boldsymbol{R}^n, \tag{1.1}$$

where $a(x)$ is a propagation speed.

We make the assumptions for the interface separating two media and $a(x)$.

Let $\varphi_0(y)=a|y|$ and $\varphi(y)\in C^1(\boldsymbol{R}^{n-1}\setminus\{0\})$, where $a\geqq 0$. We assume that $\varphi(y)$ describes the interface and satisfies

$$\sum_{|\alpha|\leq 1}|y|^{|\alpha|}|\partial^\alpha(\varphi(y)-\varphi_0(y))|=O(|y|^{-\theta})\ (|y|\longrightarrow\infty), \tag{A.0}$$

for some $\theta>0$, and

$$\sum_{|\alpha|\leq 1}|y|^{|\alpha|}|\partial^\alpha\varphi(y)|=O(|y|^{-\sigma})\ (|y|\longrightarrow 0). \tag{A.1}$$

where $0<\sigma<1/2$. For $\varphi(y)$, we use the following notation:

$$\Omega_+=\{x=(y, z):\ z>\varphi(y)\},$$

$$\Omega_-=\{x=(y, z):\ z<\varphi(y)\},$$

$$S=\{x=(y, z):\ z=\varphi(y)\}.$$

We denote the unit normal vector at the point $x\in S$ by $\nu=(\nu_1, \nu_2, \cdots, \nu_z)$ with $\nu_z>0$.

The propagation speed $a(x)>0$ is assumed to satisfy the following: for

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