TSUKUBA J. MATH.
Vol. 17 No. 1 (1993), 251—265

A METATHEORY OF NONSTANDARD ANALYSIS

By

Mariko YASUGI, Yoshiki TSUJII and Takakazu MORI

In the previous work [11] of Yasugi, we set up a formal system IR of infinitary logic, by whose proof-theoretical properties the infinitesimal calculus can be justified. Such an attempt was started due to the first author's wish to single out the essence of the metatheory of nonstandard mathematics as a "trick of the language", and was concluded with the "linkage principles" which support the nonstandard theory.

We have since pushed that thought forward and extended the result to general nonstandard analysis, with the theory of Loeb measure in mind.

Let V be the universe of analysis with the real numbers as individuals and let C be the set of constants representing V. Let A be a collection of set theoretical axioms (those necessary for analysis) and let B be a set of mathematical axioms, which will be expressed in terms of "elementary" formulas involving the constants in C. Let C be the collection of specification axioms on the domains:

$$\forall x \in d \vee [c \in d](x=c),$$

where $\vee[c \in d]$ expresses the disjunction over the domain d.

With the axioms in C, the elementary quantifiers and the "restricted" conjunctions and disjunctions becomes equivalent:

$$\forall x \in d F(x) \longleftrightarrow \wedge[c \in d] F(c).$$

C specifies the mathematical objects to the "standard" ones.

Our basic logic is an infinitary logic with "elementary" quantifiers. The "standard" analysis SA is a consequence of A and B, and C regulates the meaning of quantifiers. If one lifts the regulation C, then we obtain the subsystem GA, in which the existence of nonstandard objects will become consistent. The "internality" is characterized by "elementariness". The bridge between SA and GA is the first group of "linkage" principles stated in § 3. See also § 3 of [11]. By virtue of these principles, we can define a system NA, nonstandard analysis, as an enlargement of GA. In order to develop ex-

Received December 6, 1991.