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AUTOMORPHISMS WITH FIXED POINTS AND WEIERSTRASS POINTS OF COMPACT RIEMANN SURFACES

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§ 0. Introduction.

Let M be a compact Riemann surface of genus $g \geqq 2$ and T be a conformal automorphism of order N with t fixed points. We denote $\langle T \rangle$ the cyclic group generated by T and $M/\langle T \rangle$ the surface by identifying the equivalent points on M under the elements of $\langle T \rangle$. M is considered as a covering surface of $M/\langle T \rangle$ and the behavior of ramifications depends on the gap sequences of the fixed points.

Lewittes [7] proved that if $t \geqq 5$, then every fixed point of T is 1-Weierstrass point, and Guerrero [4] proved that if $t=1$ and the fixed point is not a 1-Weierstrass point, then T has order 6, $g \equiv 1 \pmod 6$ and the fixed point is a q-Weierstrass point for all $q \geqq 2$. Guerrero also gave examples of Riemann surfaces with automorphisms of prime order N whose two fixed points are not q-Weierstrass points. Furthermore several authors considered some cases for the relation of the fixed points and q-Weierstrass points.

Duma [2] proved that if $N=2$ and $t \geqq 3$, then every fixed point of T is a q-Weierstrass point for all $q \geqq 2$, and that if $N=3$ and $t \geqq 3$, then every fixed point of T is a q-Weierstrass point for $q \geqq 2$ ($q \neq 2 \pmod 3$). Farkas and Kra [3] proved that if T is of prime order N and $t \geqq 3$, then every fixed point is a q-Weierstrass point for $q \geqq 2$ ($q \equiv 1 \pmod N$). Accola [1] proved that if T is of prime order N and $t \geqq 3$, then every fixed point is a N-Weierstrass point. Recently Horiuchi and Tanimoto [5] gave a sufficient condition for fixed points to be q-Weierstrass point ($q \geqq 2$) and showed that the results mentioned above are obtained by using the condition and studied the case where $t \geqq 3$ and T is of order 5.

Almost all of the results mentioned above, however, are obtained under the condition that T is of prime order. In this paper we investigate the properties of automorphisms without the condition that T is of prime order. In the first

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