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REDUCTION TECHNIQUES FOR HOMOLOGICAL CONJECTURES

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Let A be a finite-dimensional k-algebra over an algebraically closed field k. We denote by mod A the category of finitely generated left A-modules. For an A-module ${}_AX$ we denote by $\mathrm{pd}_A X$ (resp. $\mathrm{id}_A X$) the projective (resp. injective) dimension of X. With $D=\mathrm{Hom}_k(-, k)$ we denote the standard duality with respect to the ground field. Then ${}_AD(A_A)$ is an injective cogenerator for mod A. To formulate some of the homological conjectures we need some more notation. Let ${}_A\mathcal{J}\subset \mathrm{mod}\, A$ be the full subcategory containing the finitely generated injective A-modules. Let $K^b({}_A\mathcal{J})$ be the homotopy category of bounded complexes over ${}_A\mathcal{J}$. Let $D^b(A)$ be the derived category of bounded complexes over mod A. We consider $K^b({}_A\mathcal{J})$ as a full subcategory of $D^b(A)$. We define.

$$K^b({}_A\mathcal{J})^{\perp}=\{X\in D^b(A)\mid \mathrm{Hom}(I, X)=0 \text{ for all } I\in K^b({}_A\mathcal{J})\}.$$

We are interested in the following conjectures:

(1) **Finitistic Dimension Conjecture:** $\mathrm{fd}(A)=\sup\{\mathrm{pd}_A X\mid \mathrm{pd}_A X<\infty\}$ is finite.

(2) **Vanishing Conjecture:** $K^b({}_A\mathcal{J})^{\perp}=0$.

(3) **Generalized Nakayama Conjecture:** For a simple module ${}_AS$ there is $i\geqq 0$ such that $\mathrm{Ext}_A^i({}_AD(A_A), {}_AS)\neq 0$.

We refer to [**AR**], [**B1**], [**H3**] and [**J**] for some further information about these conjectures.

The aim of this article is to show that using the language of triangulated categories certain reduction techniques can be obtained. To be more precise we will show the following results:

A module $T\in \mathrm{mod}\, A$ is called a (generalized) *tilting module* if the following conditions are satisfied:

(i) $\mathrm{pd}_A T<\infty$

(ii) $\mathrm{Ext}_A^i(T, T)=0$ for all $i>0$

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