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ON THE COMPLETE RELATIVE COHOMOLOGY OF FROBENIUS EXTENSIONS

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Introduction.

Let Λ be an algebra over a commutative ring K and Γ a subalgebra. Suppose that the extension Λ/Γ is a Frobenius extension. Then in [3, section 3], the complete relative cohomology group $H^{r}_{(\Lambda,\Gamma)}(M, -)$ is introduced for an arbitrary left Λ-module M and $r\in\boldsymbol{Z}$. We denote the opposite rings of Λ and Γ by Λ^{0} and Γ^{0} respectively. Put $P=\Lambda\otimes_{K}\Lambda^{0}$ and let S denote the natural image of $\Gamma\otimes_{K}\Gamma^{0}$ in P. Then the extension P/S is also a Frobenius extension. Since Λ is a left P-module with the natural way, we have $H^{r}_{(P,S)}(\Lambda, -)$. We will denote this $H^{r}_{(P,S)}(\Lambda, -)$ by $H^{r}(\Lambda, \Gamma, -)$ for [6, section 3]. In this paper, we will study this complete relative cohomology $H(\Lambda, \Gamma, -)$. In section 1, we will study relative complete resolutions of Λ and in section 2, we will introduce the dual of the fundamental exact sequence of [4, Proposition 1 and Theorem 1] for complete relative cohomology groups. In section 3, we will study an internal product like as in [9, section 2] which we will call the cup product. If the basic ring of the Frobenius extension is commutative, the cup product in this paper coincides with the product $\vee$ in [2, Exercise 2 of Chapter XI] for dimension>0.

1. Relative complete resolutions.

Let P be a ring and S a subring such that the extension P/S is a Frobenius extension. In [3], the complete (P, S)-resolution of a left P-module M is introduced. It is a (P, S)-exact sequence $\cdots\rightarrow X_{1}\xrightarrow{d_1}X_{0}\xrightarrow{d_0}X_{-1}\xrightarrow{d_{-1}}\cdots$ such that X_{n} is (P, S)-projective for all $n\in\boldsymbol{Z}$ and there exist a P-epimorphism $\varepsilon: X_{0}\rightarrow M$ and a P-monomorphism $\eta: M\rightarrow X_{-1}$ which satisfy $\eta\circ\varepsilon=d_{0}$, that is, the complete (P, S)-resolution of M is an exact sequence which consists of a (P, S)-projective resolution and a (P, S)-injective resolution of M since (P, S)-projectivity is equivalent to (P, S)-injectivity. Note that any two complete (P, S)-resolutions

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