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PRODUCT THEOREMS IN DIMENSION THEORY

Dedicated to Professor Y. Kodama on the occasion of his 60th birthday

By

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Introduction.

Throughout this paper we assume that all spaces are just topological spaces, otherwise specified. We start from the following theorem:

THEOREM 0 [10]. Let $X\times Y$ be piecewise rectangular. Then,

$$(*)\quad \mathrm{Id}(X\times Y)\leqq \mathrm{Id}\,X+\mathrm{Id}\,Y.$$

Where In Z for a space Z is a dimension function introduced by B. A. Pasynkov [9], and we will give its definition in the following section of this paper as well as the definition of piecewise rectangularity.

COROLLARY 0 [10]. *Let $X\times Y$ be normal, piecewise rectangular, and let each of X and Y satisfy a finite sum theorem for* Ind (*FST*(Ind) *for short*). *Then we have*

$$(**)\quad \mathrm{Ind}(X\times Y)\leqq \mathrm{Ind}\,X+\mathrm{Ind}\,Y.$$

The proofs for these results have not yet been published. The central ideas for those were presented by the first author at General Topology and Geometric Topology Symposium held at Tsukuba in 1990; the simplest case when $X\times Y$ is compact was talked there. Detailed proofs for Theorem 0 and Corollary 0 were given also by the first author when he visited Tsukuba in 1991 (see [12]). On this occasion we discussed the following conjecture:

CONJECTURE. Let $\Pi=X_1'\times X_2, *\in X_1'$, $X_1=X_1'\setminus\{*\}$, and the product $\Pi_0=X_1\times X_2$ be piecewise rectangular and satisfy the following condition (#).

(#) Every set H is functionally separated from $\{*\}\times X_2$ whenever H is closed in Π and $H\cap(\{*\}\times X_2)=\emptyset$. Then, we have $\mathrm{Id}\,\Pi\leqq\mathrm{Id}\,X_1'+\mathrm{Id}\,X_2$.

In this paper we shall prove this conjecture for the following cases:

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