TSUKUBA J. MATH.
Vol. 17 No. 1 (1993), 43–57

# THE PRIME $k$-TUPLETS IN ARITHMETIC PROGRESSIONS

By

Koichi KAWADA

## § 1. Introduction and notation.

In this paper we discuss a problem on the distribution of prime multiplets in arithmetic progressions. Before mentioning our problem we need to introduce the following notation. (In connection with our problem, see also the introduction of Balog's tract [1].)

For an integer $k\geqq 2$, we let $a_j(0\leqq j\leqq k-1)$ be non-zero integers, and let $b_j(0\leqq j\leqq k-1)$ be integers, and put $\boldsymbol{a}=(a_0, a_1, \cdots, a_{k-1}, b_0)$, $\boldsymbol{b}=(b_1, \cdots, b_{k-1})$, (Later, we will fix all the coordinates of $\boldsymbol{a}$, and treat an average over $\boldsymbol{b}$. This is why the unsymmetry of the definitions of $\boldsymbol{a}$ and $\boldsymbol{b}$ occurs.),

$$R(\boldsymbol{b})=R(\boldsymbol{a}, \boldsymbol{b})=\prod_{j=0}^{k-1}|a_j|\prod_{0\leq i<j\leq k-1}|a_ib_j-a_jb_i|,$$

$$N(x ; \boldsymbol{b})=N(x ; \boldsymbol{a}, \boldsymbol{b})=\{n ; 1\leqq a_jn+b_j\leqq x \text{ for all } 0\leqq j\leqq k-1\},$$

and define

$$\Psi(x ; \boldsymbol{b}, a, q)=\Psi(x ; \boldsymbol{a}, \boldsymbol{b} ; a, q)=\sum_{\substack{n\in N(x ; \boldsymbol{b}) \\ n\equiv a \pmod q}}\prod_{j=0}^{k-1}\Lambda(a_jn+b_j),$$

where $\Lambda$ denotes the von Mangoldt function. And, we let, for any prime $p$, $\rho(p)=\rho(p ; \boldsymbol{a}, \boldsymbol{b})$ be the number of solutions of the congruence

$$\prod_{j=0}^{k-1}(a_jn+b_j)\equiv 0 \pmod p,$$

and set, if $R(\boldsymbol{b})\neq 0$, $\rho(p)<p$ for all prime $p$, and $(a_ja+b_j, q)=1$ for all $0\leqq j\leqq k-1$,

$$\sigma(\boldsymbol{b} ; a, q)=\sigma(\boldsymbol{a}, \boldsymbol{b} ; a, q)=\frac{1}{q}\prod_{p\mid q}\Big(1-\frac{\rho(p)}{p}\Big)^{-1}\prod_{p}\Big(1-\frac{\rho(p)}{p}\Big)\Big(1-\frac{1}{p}\Big)^{-k}$$

and $\sigma(\boldsymbol{b} ; a, q)=0$ otherwise. Further, we put

$$Z(x)=Z(x ; \boldsymbol{a})=\{\boldsymbol{b} ; |N(x ; \boldsymbol{b})|\neq 0\},$$

where $|N(x ; \boldsymbol{b})|$ denote the length of the interval $N(x ; \boldsymbol{b})$.

Received December 11, 1991, Revised April 15, 1992.