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LOCAL INJECTIVITY OF PRYM MAPS FOR SOME FAMILIES OF COMPACT RIEMANN SURFACES

By

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Introduction.

In this paper we consider some families of double coverings of compact Riemann surfaces (or complete irreducible non-singular algebraic curves over $\mathbf{C}$) allowing ramifications, and we study the Prym varieties of these double coverings.

Let $\pi: \tilde{R}\rightarrow R$ be a double covering, where $\tilde{R}$ and R are compact Riemann surfaces of genera $\tilde{g}$ and g, and $J(\tilde{R})$ and $J(R)$ be Jacobians of $\tilde{R}$ and R, respectively. If π has $2n$ branch points, we have $\tilde{g}=2g+n-1$ by means of the Riemann-Hurwitz relation. We denote by ι the generator of the Galois group of $\tilde{R}/R$. Moreover we denote by the same ι the involution of $J(\tilde{R})$ induced by that of $\tilde{R}/R$. The norm map $Nm: J(\tilde{R})\rightarrow J(R)$ is defined by the induced map on divisor classes given by $D\rightarrow\pi(D)$ (D a divisor on $\tilde{R}$). The Prym variety $P=P(\tilde{R}/R)$ of $\tilde{R}/R$ (or $(\tilde{R}, \iota)$) is defined by the conneted component containing the origin of the kernel of Nm, and we have an isogeny $\iota_*: J(R)\times P\rightarrow J(\tilde{R})$ naturally (see Mumford [5], Fay [5], Sasaki [7]). The process taking Prym varieties defines the so-called Prym map $P: \tilde{R}/R\rightarrow P(\tilde{R}/R)$ from the family of $(\tilde{R}, \iota)$'s to the moduli space of polarized abelian varieties.

In case of unramified double coverings, Mumford [5] states some beautiful results concerning the relative dimension of the Prym map. For double coverings with ramification points, however, the contribution of those points to the Prym map might be unknown.

In this paper we will caluculate the relative dimension of the Prym map for some typical examples of $\tilde{R}/R$ with $2n$ $(n\geqq 1)$ ramification points.

We consider the following three families of compact Riemann surfaces parametrized by t or t_i's:

(I) $\tilde{R}_t: y^4=(x-1-t)(x^2+x+1)$ genus 3

$R_t: y^2=(x-1-t)(x^2+x+1)$ genus 1

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