TSUKUBA J. MATH.
Vol. 16 No. 2 (1992), 545–550

RECOLLEMENT AND IDEMPOTENT IDEALS

Dedicated to Professor Hiroyuki Tachikawa on his 60th birthday

By

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The notion of quasi-hereditary algebras was introduced by E. Cline, B. Parshall and L. Scott [3, 4, 8 and 9]. A quasi-hereditary algebra is defined by a chain of particular idempotent ideals, and induces a sequence of recollements of their derived categories. In case A is a semiprimary ring, V. Dlab and C.M. Ringel [5] studied the notion of a quasi-hereditary ring. The notion of recollement was introduced by A.A. Beilinson, J. Bernstein and P. Deligne [2]. In [7] we studied localization of triangulated categories and derived categories, and showed that recollement is equivalent to bilocalization.

Recall that an ideal I of a ring A is called idempotent if $I=AeA$ for some idempotent e of A; in particular, I is a minimal idempotent ideal provided that e is primitive. An ideal J of A is said to be a heredity ideal of A if $J^2=J$, $J(Rad\ A)J=0$, and J_A is projective. Then, in case of A being a semiprimary ring, J is a heredity ideal if and only if there exists an idempotent e of A such that: (1) $J=AeA$; (2) $Ae\otimes_{eAe}eA\cong AeA$; (3) eAe is a semisimple ring [5, 9]. In this case, E. Cline, B. Parshall and L. Scott showed that $\{\boldsymbol{D}^b(\mathrm{Mod}\ A/AeA), \boldsymbol{D}^b(\mathrm{Mod}\ A), \boldsymbol{D}^b(\mathrm{Mod}\ eAe)\}$ is recollement [9].

In this note, we give necessary and sufficient conditions for $\{\boldsymbol{D}^b(\mathrm{Mod}\ A/AeA), \boldsymbol{D}^b(\mathrm{Mod}\ A), \boldsymbol{D}^b(\mathrm{Mod}\ eAe)\}$ to be recollement in case of A is left noetherian or semiprimary. In particular, we study when a minimal idempotent ideal satisfies recollement conditions. Throughout this note, we assume that all rings have unity and that all modules are unital. For a ring A, Mod A (resp. A-Mod) is the category of right (resp., left) A-modules, and mod A (resp., A-mod) is the category of finitely presented right (resp., left) A-modules.

The author would like to thank M. Hoshino for helpful suggestions and discussions.

THEOREM 1. *Suppose A is a left noetherian or semiprimary ring. Let e be an idempotent of A. The following assertions are equivalent*:

Received November 11, 1991.