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ON THE EXCEPTIONAL SET IN GOLDBACH'S PROBLEM

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1. Introduction.

Let $E(x)$ denote the number of even integers not exceeding x that are not representable as a sum of two primes. The Goldbach conjecture asserts $E(x) \ll 1$. Unfortunately this is far from our reach. In 1923 G.H. Hardy and J.E. Littlewood [4] showed, on assuming the extended Riemann hypothesis, that

$$E(x) \ll x^{1/2+\varepsilon} \tag{1}$$

for any $\varepsilon>0$. After the fundamental work of I.M. Vinogradov [18], several authors have unconditionally given the non-trivial bounds for $E(x)$. The best one of these is due to H.L. Montgomery and R.C. Vaughan. In 1975 they [13] showed that there exists a positive constant δ such that

$$E(x) \ll x^{1-\delta}.$$

Chen J.-r. [2] gave an explicit value of δ, which is very small.

In 1973 K. Ramachandra [16] proved that, for any $A>0$,

$$E(x+x^{\theta})-E(x) \ll x^{\theta}(\log x)^{-A} \tag{2}$$

providing

$$\frac{7}{12}<\theta \leqq 1. \tag{3}$$

This bound 7/12 comes from a zero density estimate for the Dirichlet L-series. In 1981 Lou S.-t. and Yao Q. [9] attempted to sharpen the inequality (2). Later Yao [20] replaced, in the same range of θ as (3), the right hand side of (2) by $x^{\theta(1-\delta)}$ with some $\delta>0$.

It is of some interest, from the point of veiw of (1), to demonstrate the formula (2) for θ less than 1/2. We shall present such a result.

THEOREM. *Let $A>0$ and $7/48<\theta\leqq 1$ be given. Then we have*

$$E(x+x^{\theta})-E(x) \ll x^{\theta}(\log x)^{-A}$$

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