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WEAKLY NORMAL FILTERS AND LARGE CARDLINALS

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0. Introduction.

In this paper, κ denotes an uncountable regular cardinal and λ a cardinal $\geqq\kappa$. For any such pair, $P_{\kappa}\lambda$ is the set $\{x\subset\lambda: |x|<\kappa\}$.

An "ideal on $P_{\kappa}\lambda$" is always a "proper, nonprincipal, κ-complete, fine ideal on $P_{\kappa}\lambda$" unless specified. (An ideal I is fine if for all $\alpha<\lambda$, $\{x\in P_{\kappa}\lambda: \alpha\notin x\}\in I$.) For any ideal I, $I^{+}=P(P_{\kappa}\lambda)-I$ and I^{*} is the filter dual to I.

DEFINITON. An ideal I as well as I^{*} are said to be *weakly normal* iff for every regressive function $f: P_{\kappa}\lambda\rightarrow\lambda$,

$$(\exists\gamma<\lambda)(\{x\in P_{\kappa}\lambda: f(x)<\gamma\}\in I^{*}).$$

The above definition is a translation of Kanamori's "weak normality" for filters on κ in [**5**]. There is another weak normality presented by Mignone [**10**]. It is known that our notion is Mignone's weak normality plus some saturation property and every $cf\lambda$-saturated normal ideal on $P_{\kappa}\lambda$ has our weak normality.

κ is said to be λ-compact if there is a fine ultrafilter on $P_{\kappa}\lambda$. If κ is λ-compact, $P_{\kappa}\lambda$ carries many fine ultrafilters. Moreover every fine ultrafilter has a weakly normal fine ultrafilter which is Rudin-Keisler ordering below it. So, it may be a natural question whether κ is large if a weakly normal filter on $P_{\kappa}\lambda$ exists.

In §1, we consider a case where one can say κ is large.

Kunen-Paris [**7**] and Kunen [**6**] consider the possibility of $S(\kappa, \eta)$ holding for various κ, η where

$S(\kappa, \eta)\equiv$There is a κ-complete η-saturated ideal on κ. §2 is devoted to an application of their methods to weakly normal ideals on $P_{\kappa}\lambda$.

Much of our notation is standard, and Jech [**4**] or [**7**] should be consulted.

§1. κ may be strong compact.

For a reader's convenience, we give a proof of a lemma which appears

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