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A COMBINATORIAL PROOF FOR ARTIN'S PRESENTATION OF THE BRAID GROUP B_n AND SOME CYCLIC ANALOGUE

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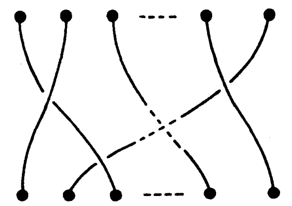
1. Artin's presentation.

For each $n\geqq 1$, let S_n be the symmetric group on n letters $\{1, 2, \cdots, n\}$, and B_n the geometric braid group with n strings.



There is a natural homomorphism, called χ_n, of B_n onto S_n. As usual, S_{n-1} and B_{n-1} are regarded as subgroups of S_n and B_n respectively, and then the restriction of χ_n to B_{n-1} coinsides with χ_{n-1}. Put $B_n^0=\chi_n^{-1}(S_{n-1})$. Then B_{n-1} is a subgroup of B_n^0.

Let $\tilde{B}_n$ be the group presented by the generators:

$$\sigma_1, \sigma_2, \cdots, \sigma_{n-1}$$

and the defining relations:

$$\begin{cases} \sigma_i\sigma_j\sigma_i=\sigma_j\sigma_i\sigma_j & \text{if } |i-j|=1\,; \\ \sigma_i\sigma_j=\sigma_j\sigma_i & \text{if } |i-j|\neq 0, 1\,. \end{cases}$$

Put

$$\tau_i=\sigma_{n-1}^{-1}\cdots\sigma_{i+1}^{-1}\sigma_i^2\sigma_{i+1}\cdots\sigma_{n-1} \qquad \text{for } 1\leqq i\leqq n-2\,,$$

$$\tau_{n-1}=\sigma_{n-1}^2\,.$$

Let $\tilde{B}_n^0$ be the subgroup of $\tilde{B}_n$ generated by $\sigma_1, \cdots, \sigma_{n-2}, \tau_1, \cdots, \tau_{n-1}$. Then there is a natural homomorphism of $\tilde{B}_{n-1}$ into $\tilde{B}_n^0$.

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