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A CLASS OF MULTIVALENT FUNCTIONS

By

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1. Introduction.

Let $A(p)$ be the class of functions of the form

$$f(z)=z^{p}+\sum_{n=p+1}^{\infty}a_{n}z^{n} \qquad (p\in N=1, 2, 3, \cdots)$$

which are analytic in $U=\{z \mid |z|<1\}$.

A function $f(z)\in A(p)$ is said to be p-valently starlike iff

$$\mathrm{Re}\frac{zf'(z)}{f(z)}>0 \qquad \text{in } U.$$

We denote by $S(p)$ the subclass of $A(p)$ consisting of functions which are p-valently starlike in U. Further, a function in $A(p)$ is said to be p-valently convex iff

$$1+\mathrm{Re}\frac{zf''(z)}{f'(z)}>0 \qquad \text{in } U.$$

Also we denote by $C(p)$ the subclass of $A(p)$ consisting of all p-valently convex functions in U.

MacGregor [2] investigated the class of functions which are analytic in U, $f(0)=f'(0)-1=0$ and satisfy the condition

$$|f'(z)-1|<1 \qquad \text{in } U.$$

Let F denote the class of functions which satisfy the above conditions.

MacGregor [2, Theorem 6] obtained the following result:

THEOREM A. *If $f(z)\in F$, then $f(z)$ is starlike in $|z|<\sqrt{4/4}\doteqdot 0.894$.*

Nunokawa [4] and Nunokawa, Fukui, Owa, Saitoh and Sekine [6] improved Theorem A. Mocanu [3] showed that there is a function $f(z)\in A(1)$ which is a member of F but not starlike in $|z|<1$.

THEOREM B. *If $f(z)\in F$, then $f(z)$ is starlike in $|z|<r_{1}<1$, where r_{1} is the*

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