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FOURTH ORDER SEMILINEAR PARABOLIC EQUATIONS

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1. Introduction.

The aim of this paper is to give a simple proof of the existence of a smooth solution to the semilinear parabolic equation with fourth order elliptic operator:

(1) $$u_t=-\varepsilon^2\Delta^2 u+f(t, x, u, u_x, u_{xx})=: L(t, x, u),$$

$x\in\Omega\subset R^n$, Ω is a bounded domain, $t\in[0, T_{\max})$, $T_{\max}\leqq+\infty$, where $\Delta^2=\Delta\circ\Delta$, u_x is a vector of partial derivatives $(u_{x_1}, \cdots, u_{x_n})$ and u_{xx} stands for the Hessian matrix $[u_{x_i x_j}]$, $i, j=1, \cdots, n$. We consider (1) together with initial-boundary conditions

(2) $$u(0, x)=u_0(x), \qquad x\in\Omega,$$

(3) $$\frac{\partial u}{\partial n}=\frac{\partial(\Delta u)}{\partial n}=0 \qquad \text{when } x\in\partial\Omega.$$

Schematically we may write (3) as $B_1u=B_2u=0$.

In recent years a rapidly growing interest has been evinced in special problems such as the Cahn-Hilliard or the Kuramoto-Sivashinsky equations covered by our general form (1). Recently weak solutions for these special problems were considered in Temam's monograph [12]. The methods used here are an extension of those in previous papers [5, 6] devoted to the study of second order equations. General scheme of our proof of local existence (construction of the set X, considerations following (19)) is similar to the classical proof of the Picard theorem for solutions of ordinary differential equations.

2. Motivation.

We have two tasks in this paper. In Part I we prove local in time classical solvability of (1)-(3). We cannot expect global (that is in an arbitrarily large time interval) solvability of (1)-(3) under the weak assumption of local Lipschitz continuity of the nonlinear term f only (because of the possible rapid growth

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