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ON PRIME TWINS IN ARITHMETIC PROGRESSIONS

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1. Introduction.

Let q and a be coprime positive integers. Put, for a non-zero integer k,

$$\Psi(x;q,a,2k)=\sum_{\substack{0<m,n\leq x\\ m-n=2k\\ n\equiv a(\mathrm{mod}\ q)}}\Lambda(m)\Lambda(n)$$

where Λ is the von Mangoldt function. It is expected that, provided $(a+2k,q)=1$, Ψ is asymptotically equal to

$$H(x;q,2k)=\mathfrak{S}\prod_{\substack{p|qk\\ p>2}}\left(\frac{p-1}{p-2}\right)\cdot\frac{x-|2k|}{\varphi(q)}$$

where

$$\mathfrak{S}=2\prod_{p>2}\left(1-\frac{1}{(p-1)^2}\right).$$

Let

$$E(x;q,a,2k)=\begin{cases}\Psi-H, & \text{if } (a+2k,q)=1\\ \Psi, & \text{otherwise.}\end{cases}$$

It is well known that $E(x;1,1,2k)$ is small in an averaged sense over k.

In 1961 A.F. Lavrik [5] showed that, for any $A,B>0$,

$$\sum_{0<2k\leq x}|E(x;q,a,2k)|\ll x^2(\log x)^{-A}$$

uniformly for $(a,q)=1$ and $q\leqq(\log x)^B$. Recently H. Maier and C. Pommerance considered the inequality

$$\sum_{q\leq Q}\max_{(a,q)=1}\sum_{0<2k\leq x}|E(x;q,a,2k)|\ll x^2(\log x)^{-A},$$

which may be regarded as an analogue to the Bombieri-Vinogradov theorem. They [3] showed that the above is valid for $Q\leqq x^{\delta}$ with some small $\delta>0$, and applied their formula to a problem concerned with gaps between primes. Later A. Balog [1] generalized this to the case of prime multiplets, and extended the

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