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ON THE VANISHING OF HOCHSCHILD COHOMOLOGY $H^1(\Lambda, \Lambda\otimes\Lambda)$ FOR A LOCAL ALGEBRA Λ

By

Qiang ZENG

§ 0. Introduction.

Throughout this paper we assume that Λ is a finite dimensional local algebra over an algebraically closed field K. By considering certain subgroups of the Hochschild cohomology groups of Λ-bimodule $\Lambda\otimes\Lambda$ for a generalized biserial commutative algebra Λ the author proved in [7] that Λ is selfinjective if and only if $H^1(\Lambda, \Lambda\otimes\Lambda)=0$. Here Λ is called to be *generalized biserial* if the both composition lengths of $_{\Lambda}((\mathrm{rad}\,\Lambda)^i/(\mathrm{rad}\,\Lambda)^{i+1})$ and $((\mathrm{rad}\,\Lambda)^i/(\mathrm{rad}\,\Lambda)^{i+1})_{\Lambda}$ $\leqq 2$ for all $i=1, 2, \cdots$.

On the other hand for a commutative algebra Λ with cube zero radical using Hoshino's results Asashiba proved in [1] that Λ is selfinjective if and only if $\mathrm{Ext}^1_{\Lambda}(_{\Lambda}\mathrm{Hom}_K(\Lambda_{\Lambda}, K), {}_{\Lambda}\Lambda)\cong H^1(\Lambda, \Lambda\otimes\Lambda)=0$.

One of the purposes of this paper is to show in § 1 that Asashiba's results together with Hoshino's can be proved directly by calculating the similar subgroups of the Hochschild cohomology of Λ-bimodule $\Lambda\otimes\Lambda$ with [7].

It was conjectured in [5] that Λ is selfinjective if $H^i(\Lambda, \Lambda\otimes\Lambda)=0$ for $i=1, 2, \cdots$. The above results implies that a commutative algebra Λ is selfinjective if $H^1(\Lambda, \Lambda\otimes\Lambda)=0$ and Λ is either generalized biserial or of cube zero radical. So it is interesting to consider the same problem for an algebra with quartic zero radical which is a homomorphic image of the polynomial ring $K[x, y]$ of variables x and y. In § 2 we shall prove that for such algebras we have also an affirmative answer. However it is to be noted here that for this case it needs to consider the larger subgroups of the Hochschild cohomology of Λ-bimodule $\Lambda\otimes\Lambda$ different than ones for the above stated cases.

As was seen in [6] and [7] for commutative algebra Λ it holds that the both composition lengths of $(\mathrm{rad}\,\Lambda)/(\mathrm{rad}\,\Lambda)^2$ and $(\mathrm{rad}\,\Lambda)^2/(\mathrm{rad}\,\Lambda)^3\leqq 2$ implies that Λ is generalized biserial. In § 3 we shall show that we can generalize the above fact for non-commutative algebras. At the end of this section we shall quote that for a (not necessarily commutative) positively **Z**-gradable algebra Λ

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