TSUKUBA J. MATH.
Vol. 16 No. 2 (1992), 295—313

EXISTENCE RESULTS FOR QUASILINEAR DIRICHLET PROBLEM

By

J. CHABROWSKI

1. Introduction.

This paper deals with the Dirichlet problem

$$-\sum_{i,j=n}^{n} D_j(a_{ij}(x, u)D_i u)+c(x)u=b(x, u, Du) \quad \text{in } Q, \tag{1}$$

$$u(x)=\phi(x) \quad \text{on } \partial Q, \tag{2}$$

in a bounded domain $Q\subset \boldsymbol{R}_n$ with the boundary ∂Q of class C^2 and a function ϕ which, in general, is not a trace of an element from the space $W^{1,2}(Q)$. We consider two cases: $\phi\in L^{\infty}(\partial Q)$ (Section 3 and 4) and $\phi\in L^2(\partial Q)$ (Section 5).

In case where $\phi\in L^{\infty}(\partial Q)$ we establish some existence theorems for the problem (1), (2) under the assumption that the nonlinearity $b(x, u, p)$ grows quadratically in p. In recent years the problem (1), (2), with the nonlinearity b growing quadratically in p, has attracted some interest (see [1], [2], [7] and the references given there). In paper [1] the existence result was established in the space $\mathring{W}^{1,2}(Q)\cap L^{\infty}(Q)$ (that is, $\phi\equiv 0$ on ∂Q). The results of [2] show that under suitable assumptions on $b(x, u, p)$ one can also obtain unbounded solutions in $\mathring{W}^{1,2}(Q)$. The use of a weighted Sobolev space in [7] allowed one to obtain an existence theorem for the problem (1), (2) with $\phi\in L^{\infty}(\partial Q)$. In the case where $\phi\in L^2(\partial Q)$, we assume that the nonlinearity has a linear growth in p. The present paper is a generalization of [7].

The paper is organized as follows. In Section 2 we assemble definitions, assumptions and some terminology adopted in this work. Lemma 1, proved in this section, justifies our approach to the problem (1), (2) with the nonlinearity growing quadratically in p. Section 3 contains the main existence result of this paper which is closely related to Theorem 2.1 in [1] and Theorem 2 in [7]. The existence result in [1] was proved for more general quasilinear elliptic equations under the assumption of the existence of bounded sub and supersolutions but it can be applied only to the boundary data from $H^{1/2}(\partial Q)$.

Received August 12, 1991, Revised November 12, 1991.