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ON SOME STARLIKENESS CONDITIONS FOR ANALYTIC FUNCTIONS

By

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Let $A(p)$ denote the class of functions $f(z)=z^{p}+\sum_{n=p+1}^{\infty}a_{n}z^{n}$ which are analytic in the open disk $E=\{z:|z|<1\}$.

A function $f(z)\in A(p)$ is called p-valently starlike with respect to the origin iff

$$Re\frac{zf'(z)}{f(z)}>0 \quad \text{in } E.$$

We denote by $S^{*}(p)$ the subclass of $A(p)$ consisting of functions which are p-valently starlike in E.

Mocanu [3, Theorem 1] proved that if $f(z)\in A(1)$ and

$$|\arg f'(z)|<\frac{\pi}{2}\alpha_{0}=0.968\cdots, \qquad z\in E,$$

where $\alpha_{0}=0.6165\cdots$ is the unique root of the equation

$$2\tan^{-1}(1-\alpha)+\pi(1-2\alpha)=0,$$

then $f(z)\in S^{*}(1)$.

In [5], Nunokawa proved the following theorem.

THEOREM A. *Let* $p\geqq 2$. *If* $f(z)\in A(p)$ *satisfies*

$$|\arg f^{(p)}(z)|<\frac{3}{4}\pi \qquad \text{in } E,$$

then $f(z)$ *is* p-*valent in* E.

DEFINITION 1. Let $F(z)$ be analytic and univalent in E, and suppose that $F(E)=D$. If $f(z)$ is analytic in E, $f(0)=F(0)$, and $f(E)\subset D$, then we say that $f(z)$ is subordinate to $F(z)$ in E, and we write

$$f(z)\prec F(z).$$

DEFINITION 2. If the function $f(z)$ is analytic in E and if for every non-

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