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COVERINGS OVER d-GONAL CURVES

By

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§ 1. Introduction.

Let M be a compact Riemann surface and f be a meromorphic function on M. Let (f) be the principal divisor associated to f and $(f)_\infty$ be the polar divisor of f. We call f a meromorphic function of degree d if $d=\text{degree }(f)_\infty$. If d is the minimal integer in which a meromorphic function of degree d exists on M, then we call M a d-gonal curve.

Now we assume that M is d-gonal, and consider a covering map $\pi': M'\to M$ that M' still remains d-gonal. The purpose of this paper is to show how such π' can be characterized.

The case that π' is a normal covering and $d=2$ (i.e., M is hyperelliptic) has been already studied ([2], [3], [4] and [7]). In this case the existence of the hyperelliptic involution v' on M' plays an important role. More precisely, as v' commutes with each element of the Galois group $G=Gal(M'/M)$, v' induces the hyperelliptic involution v on M and we can reduce π' to a normal covering $\pi: \boldsymbol{P}_1'\to\boldsymbol{P}_1$ with Galois group G, where $\boldsymbol{P}_1'$ and $\boldsymbol{P}_1$ are Riemann spheres isomorphic to quotient Riemann surfaces $M'/\langle v'\rangle$ and $M/\langle v\rangle$ respectively. On the other hand it is known that finite subgroups of the linear transformation group are cyclic, dihedral, tetrahedral, octahedral and icosahedral. Horiuchi [3] decided all the different normal coverings $\pi': M'\to M$ over a hyperelliptic curve M that M' still remains a hyperelliptic curve by investigating each of above five types.

Let M be a d-gonal curve. In this paper we will show at first that a covering map $\pi': M'\to M$ (not necessarily normal) with d-gonal M' canonically induces some covering map $\pi: \boldsymbol{P}_1'\to\boldsymbol{P}_1$ (Theorem 2.1 § 2). Moreover if both M and M' have unique linear system g_d^1 and π' is normal, then we can see that π is also normal (Cor. 2.3).

In § 3, § 4 and § 5 we assume that M is a cyclic p-gonal curve for a prime number p. We will determine all ramification types of normal coverings $\pi': M'\to M$ with p-gonal M' by the same way as Horiuchi did in case $p=2$(§ 4),

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