

UNIVERSAL SPACES FOR SOME FAMILIES OF RIM-SCATTERED SPACES

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1. Introduction.

1.1. Definitions and notations. All spaces considered in this paper are separable and metrizable and the ordinals are countable.

Let F be a subset of a space X . By $Bd(F)$, $Cl(F)$, $Int(F)$ and $|F|$ we denote the boundary, the closure, the interior and the cardinality of F , respectively. An open (respectively, closed) subset U of X' is called *regular* iff $U = Int(Cl(U))$ (respectively, $U = Cl(Int(U))$). If X is a metric space, then the diameter of F is denoted by $diam(F)$. A map f of a space X into a space Y is called *closed* iff the subset $f(F)$ of Y is closed for every closed subset F of X .

A *compactum* is a compact metrizable space; a *continuum* is a connected compactum. A space is said to be *scattered* iff every non-empty subset has an isolated point.

A space Y is said to be an *extension* of X iff X is a dense subset of Y . A space Y is said to be a *compactification* of X iff Y is a compact extension of X . Let Y and Z be extensions of X . A map π of Y into Z is called a *natural projection* iff $\pi(x) = x$ for every $x \in X$. Obviously, if there exist a natural projection of Y into Z , then it is uniquely determined.

A space T is said to be *universal* for a family A of spaces iff both the following conditions are satisfied: (α) $T \in A$, (β) for every $X \in A$, there exists an embedding of X in T . If only condition (β) is satisfied, then T is called a *containing space* for a family A .

A *partition* of a space X is a set D of closed subsets of X such that (α) if $F_1, F_2 \in D$ and $F_1 \neq F_2$, then $F_1 \cap F_2 = \emptyset$, and (β) the union of all elements of D is X . The *natural projection* of X onto D is the map π defined as follows, if $x \in X$, then $\pi(x) = F$, where F is the uniquely determined element of D containing x . The *quotient space* of the partition D is the set D with a topology which is the maximal on D for which the map π is continuous. (We observe that we use the same notation for a partition of a space and for the correspond-