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ON REAL HYPERSURFACES OF A COMPLEX PROJECTIVE SPACE II

Dedicated to Professor Mikio NAKAMURA on his retirement from Kumamoto University, College of Medical Science

By

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§ 0. Introduction

Let $P_n(C)$ be an n-dimensional complex projective space with Fubini-Study metric of constant holomorphic sectional curvature 4, and let M be a real hypersurface of $P_n(C)$. M has an almost contact metric structure (ϕ, ξ, η, g) induced from the complex structure J of $P_n(C)$ (see, § 1). We denote by A, R, S, the shape operator, the curvature tensor and the Ricci tensor of type (1, 1) on M, respectively. Many differential geometers have studied M (cf. [1], [6], [9], [11] and [12]) by using the structure (ϕ, ξ, η, g).

Typical examples of real hypersurfaces in $P_n(C)$ are homogeneous ones. R. Takagi ([10]) showed that all homogeneous real hypersurfaces in $P_n(C)$ are realized as the tubes of constant radius over compact Hermitian symmetric spaces of rank 1 or rank 2. Namely, he showed the following:

THEOREM T ([10]). *Let M be a homogeneous real hypersurface of $P_n(C)$. Then M is a tube of radius r over one of the following Kaehler submanifolds:*

(A_1) *hyperplane $P_{n-1}(C)$, where $0<r<\pi/2$,*

(A_2) *totally geodesic $P_k(C)$ $(1\leqq k\leqq n-2)$, where $0<r<\pi/2$,*

(B) *complex quadric Q_{n-1}, where $0<r<\pi/4$,*

(C) *$P_1(C)\times P_{(n-1)/2}(C)$, where $0<r<\pi/4$ and $n(\geqq 5)$ is odd,*

(D) *complex Grassmann $G_{2,5}(C)$, where $0<r<\pi/4$ and $n=9$,*

(E) *Hermitian symmetric space $SO(10)/U(5)$, where $0<r<\pi/4$ and $n=15$.*

Due to his classification, we find the number of distinct constant principal curvatures of a homogeneous real hypersurface is 2, 3 or 5. Here note that the vector ξ of any homogeneous real hypersurface M (which is a tube of radius r) is a principal curvature vector with principal curvature $\alpha=2\cot 2r$ with multiplicity 1 (for further details, see [11]).

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