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TILTING LATTICES OVER ORDERS ASSOCIATED WITH SIMPLE MODULES

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Let R be a complete discrete valuation ring with quotient field K and Λ a basic R-order in a separable K-algebra, and let e be a primitive idempotent of Λ. In this paper we shall study tilting Λ-lattices in the form of $T=(1-e)\Lambda \oplus \mathrm{Tr}_L(Je)$ where J is the Jacobson radical of Λ and Tr_L is the transpose functor for Λ-lattices.

Tilting theory was initiated by Brenner and Butler [5] and its general theory over artin algebras was given in Happel and Ringel [6] and Bongartz [4] and has been used and developed by many authors not only in the study of representations of artin algebras but also in more general situations. Among them tilting modules arising from suitable simple modules are concrete and typical ones (see [2] and [5]). While almost all general results in [4] are reconstructed in the case of orders by Roggenkamp [8], it seems also to be desirable to provide an order version of such tilting modules and study its fundamental properties, which is the aim of this paper.

In Section 1, we shall recall some definitions and notation which will be used throughout the paper. In Section 2, we shall show that $T=(1-e)\Lambda \oplus \mathrm{Tr}_L(Je)$ is a tilting Λ-lattice if and only if Je is not Λ-reflexive and Λe is not isomorphic to a direct summand of the projective cover of Je (Theorem 2.1). We call such a tilting Λ-lattice Brenner-Butler type (BB-*type* for short). We shall also show that T is a tilting Λ-lattice of BB-type if and only if T is a tilting left Γ-lattice of BB-type and $\Lambda \cong \mathrm{End}_\Gamma(T)$ where $\Gamma=\mathrm{End}_\Lambda(T)$ (Theorem 2.4). As an application of Theorem 2.1 we shall show that a non-hereditary, basic tiled R-order of finite global dimension always has tilting lattices of BB-type (Proposition 2.5). As a special class of BB-type, in Section 3, we shall introduce the notion of tilting lattices of Auslander-Platzeck-Reiten type (APR-*type* for short), which arise from almost split sequences starting from certain projective modules. It should be noted that in the case of orders we cannot consider simple projective modules. We shall replace simplicity by injectivity of its radical. (See Theorem 3.1.) In Section 4, we shall precisely describe the

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