

LATTICE-VALUED REPRESENTATION OF THE CUT-ELIMINATION THEOREM¹⁾

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In 1934 G. Gentzen [1] presented the first order classical and intuitionistic predicate calculi LK and LJ , and expressed and proved his Hauptsatz or the cut-elimination theorem for them. In 1953 G. Takeuti [11] announced the fact that his fundamental conjecture or the cut-elimination theorem for his GLC implies finitistically the consistency of analysis, where GLC is a simple type theory formulated analogously to LK . From that time on he has proved successively but constructively that the fundamental conjecture is true for many subsystems of GLC .

In 1967 M. Takahashi [9] gave a general affirmative solution to Takeuti's fundamental conjecture by means of non-constructive methods (see also [10]). Takahashi's proof based on a result of K. Schütte [7] and previously W. Tait [8] had proved the cut-elimination theorem for second order predicate logic. In 1971 G. Y. Girard [2], for the intuitionistic GLC , gave a syntactical cut-elimination procedure and proved the finiteness of the procedure by use of non-constructive arguments but by no use of the law of excluded middle.

Gentzen [1] says his Hauptsatz had been found originally for the natural intuitionistic calculus NJ , that is a first order intuitionistic system of natural deductins given in [1], but he did not discourse in detail. In 1965 D. Prawitz [5] formulated the Hauptsatz or his normal form theorem for NJ ²⁾ (and for a classical natural deduction system admitting no disjunctions nor existential quantifications). There are several studies of the normal form theorem for higher order natural deduction systems: Prawitz [6], P. Martin-Löf [3], [4], and so on.

In this paper, as our Main Theorem, we shall give a semi-algebraic representation of the cut-elimination theorem. No concrete cut-elimination procedure

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¹⁾ This paper was read at a Symposium, 27–29 November 1989, to commemorate the 80th birthday of Prof. Katuzi Ono, Professor Emeritus of Nagoya University.

²⁾ The author attended Prof. Ono's lecture concerning his formulation of the normal form theorem for NJ at a Logic Symposium, 15–18 October 1966, Chiba. At that time, none of the participants knew the Prawitz formulation.