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A FACTOR OF SINGULAR HOMOLOGY

By

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0. Introduction

Singular homology is a beautiful theory, in which we can see a clear correspondence between Algebra and Topology. However, it behaves badly on topological spaces which are not locally simply connected in comparison with Čech homology. In the present paper we introduce a canonical factor $H_n^T(X)$ of singular homology $H_n(X)$, which agrees with singular homology on ANR's and behaves well on the indicated spaces. We also introduce a notion "quasi-homotopy" for continuous maps. It turns out that the factor is invariant under quasi-homotopy.

We state definitions and basic facts in Section 1. In Section 2, we prove that $H_n^T(X)=H_n(X)$ for every ANR X. In Section 3, we show that $H_0^T(X)$ is isomorphic to a free abelian group whose rank is equal to the cardinality of equivalence classes with respect to a certain kind of connectedness. There we also introduce a notion "quasi-homotopy" and show that the factor is invariant under quasi-homotopy. In Sections 4, 5 and 6, one can see the advantage of $H_n^T(X)$ to the singular homology groups $H_n(X)$. More precisely, $H_n^T(X)$ are calculated for certain spaces such as the so-called Hawaiian earring and infinite products and so on. Furthermore, certain natural abelian groups are realized as $H_n^T(X)$ by natural topological spaces X. We shall show that any homomorphism from $H_n^T(X)$ to $H_n^T(Y)$ is induced by a continuous map if X and Y are obtained by attaching copies of $\boldsymbol{S}^n$ in certain ways. For example, for the Hawaiian earring $\boldsymbol{H}$, any homomorphism from $H_1^T(\boldsymbol{H})$ to itself can be induced by a continuous map from $\boldsymbol{H}$ to itself, though this fact does not hold for $H_1(\boldsymbol{H})$. We also characterize slenderness of abelian groups by using $H_1^T(\boldsymbol{H})$ and the notion of spatial homomorphism. (See Theorems 6.2, 6.3 and Remark 6.4.) Ralph [22] has defined a factor of singular chain and homology groups HA and HM. In Section 4, one can see that H_n^T has similar effect as HA.

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