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SOBOLEV SPACES IN THE GENERALIZED DISTRIBUTION SPACES OF BEURLING TYPE

By

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1. Introduction

In this paper we extend the concept of Sobolev spaces to the generalized distribution spaces of Beurling type and investigate the Sobolev imbedding theorem, the Rellich's compactness theorem and etc on these generalized Sobolev spaces.

For this purpose we briefly introduce the basic spaces and theories which we need in this paper. The reader can find the details in [3]. Let $\mathcal{M}_c$ the set of all continuous real valued functions ω on R^n which satisfies the following conditions:

(α) $0=\omega(0)\leqq\omega(\xi+\eta)\leqq\omega(\xi)+\omega(\eta)$, ξ, $\eta\in R^n$.

(β) $\int_{R^n}\frac{\omega(\xi)}{(1+|\xi|)^{n+1}}d\xi<\infty$

(γ) $\omega(\xi)\geqq a+b\log(1+|\xi|)$ for some constants a and $b>0$.

(δ) $\omega(\xi)$ is radial.

With the weight functions ω in $\mathcal{M}_c$ and open set Ω in R^n Björck defines $\mathcal{D}_\omega(\Omega)$ the set of all ϕ in $L^1(R^n)$ such that ϕ has compact support in Ω and

$$\|\varphi\|_\lambda=\int_{R^n}|\hat{\phi}(\xi)|e^{\lambda\omega(\xi)}d\xi<\infty$$

for all $\lambda>0$. The space $\mathcal{D}_\omega(\Omega)$ equipped with the inductive limit topology, as $\mathcal{D}(\Omega)$, is Fréchet and we call $\mathcal{D}'_\omega(\Omega)$, the dual of $\mathcal{D}_\omega(\Omega)$, the Beurling's generalized distribution space. They denote by $\mathcal{E}_\omega(\Omega)$ the set of all complex valued functions ψ in Ω such that $\psi\phi\in\mathcal{D}_\omega(\Omega)$ for all $\phi\in\mathcal{D}_\omega(\Omega)$ and the topology is given by the semi-norms $\|\phi\psi\|_\lambda$ for every $\lambda>0$ and every ϕ in $\mathcal{D}_\omega(\Omega)$. The dual space $\mathcal{E}'_\omega(\Omega)$ of the space $\mathcal{E}_\omega(\Omega)$ can be identified with the set of all elements of $\mathcal{D}'_\omega(\Omega)$ which have compact support contained in Ω. They also extend the Schwartz space denoting by $\mathcal{S}_\omega$ the space of all C^∞-function ϕ in $L^1(R^n)$ with the property that for each multi-index α and each non-negative number λ we have

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