TSUKUBA J. MATH.
Vol. 15 No. 1 (1991), 185—192

A REMARK ON ARTIN-SCHREIER CURVES WHOSE HASSE-WITT MAPS ARE THE ZERO MAPS

By

Susumu IROKAWA and Ryuji SASAKI

1. Introduction

Let X be a complete non-singular algebraic curve over an algebraically closed field k of positive characteristic p. Let $F:\mathcal{O}_X\rightarrow\mathcal{O}_X$ be the Frobenius homomorphism $F(\alpha)=\alpha^p$, and denote the induced p-linear map $H^1(X, \mathcal{O}_X)\rightarrow H^1(X, \mathcal{O}_X)$ again by F, which is called the Hasse-Witt map. The dimension of the semi-simple subspace $H^1(X, \mathcal{O}_X)_s$ of $H^1(X, \mathcal{O}_X)$ is denoted by $\sigma(X)$ and called the p-rank of a curve X, which is equal to the p-rank of the Jacobian variety of X.

Let $\pi: X\rightarrow Y$ be a p-cyclic covering of complete non-singular curves over k. Then the Deuring-Šafarevič formula is the following:

$$\sigma(X)-1+r=p(\sigma(Y)-1+r) \tag{1.1}$$

where r is the number of the ramification points with respect to π (see Subrao [10], Deuring [3], Šafarevič [8]).

An algebraic curve X, which is not birationally equivalent to $\boldsymbol{P}^1$, is called an Artin-Schreier curve if there is a p-cyclic covering $\pi: X\rightarrow\boldsymbol{P}^1$. Then the p-rank $\sigma(X)$ of X is immediately known by the above formula, however the rank of the Hasse-Witt map is not known. In this article, we shall prove the following.

THEOREM. *Let X be an Artin-Schreier curve defined over an algebraically closed field k, of positive characteristic p. Then the Hasse-Witt map of X is the zero map if and only if X is birationally equivalent to the complete non-singular algeraic curve defined by the equation*

$$y^p-y=x^l$$

for some divisor l ($l\geqq 2$) of $p+1$.

The Jacobian variety of a curve X is isomorphic to the product of super-singular ellitic curves if and only if the Cartier operator is the zero map

Received March 1, 1990. Revised May 8, 1990.