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ON PRIME TWINS

In honorem Professoris Saburô Uchiyama annos LX nati

By

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1. Introduction.

It has long been conjectured that there exist infinitely many prime twins. There is even the hypothetical asymptotic formula for the number of prime pairs. Let

$$\Psi(y, 2k)=\sum_{2k<n\leqq y}\Lambda(n)\Lambda(n-2k)$$

where Λ is the von Mangoldt function, then it is expected that

$$(*)\qquad \Psi(y, 2k)\sim\mathfrak{S}(2k)(y-2k) \quad \text{as} \quad y\to\infty$$

with

$$\mathfrak{S}(2k)=2\prod_{p>2}\left(1-\frac{1}{(p-1)^2}\right)\prod_{\substack{p\mid k\\ p>2}}\left(\frac{p-1}{p-2}\right).$$

No proof of these has ever been given.

But it is well known that the above $(*)$ is valid for almost all $k\leqq y/2$. Recently, D. Wolke [4] has refined this classical result. He showed that in the range

$$2x\leqq y\leqq x^{8/5-\varepsilon}, \qquad \varepsilon>0,$$

the formula $(*)$ holds true for almost all $k\leqq x$. Moreover he remarked that, on assuming the density hypothesis for L-series, the exponent 8/5 may be replaced by 2.

In the present paper we shall improve this exponent beyond 2.

THEOREM. *Let* ε, A *and* $B>0$ *be given and*

$$2x\leqq y\leqq x^{3-\varepsilon}.$$

Then, except possibly for $O(x(\log x)^{-A})$ *integers* $k\leqq x$, *we have*

$$\Psi(y, 2k)=\mathfrak{S}(2k)(y-2k)+O(y(\log y)^{-B})$$

where the implied O*-constants depend only on* ε, A *and* B.

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