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ON THE ANTI-SELF-DUALITY OF THE YANG-MILLS CONNECTION OVER HIGHER DIMENSIONAL KAEHLERIAN MANIFOLD

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1. Introduction.

Let M be a Kaehler manifold of complex dimension $n\geqq 2$, with a Kaehler form Φ, where Φ is locally expressed by $\Phi=\sqrt{-1}\sum g_{\alpha\bar{\beta}}dz^{\alpha}\wedge d\bar{z}^{\beta}$ and a Kaehler metric $g=\sum g_{\alpha\bar{\beta}}dz^{\alpha}\otimes d\bar{z}^{\beta}$. A connection A on a principal fibre bundle $\boldsymbol{P}$ over M with the structure group G is said to be *Yang-Mills* when it gives a critical point of the Yang-Mills functional. It satisfies the Yang-Mills equation $d_A{*}F_A=0$ for the curvature F_A. Thus with the Bianchi identity $d_AF_A=0$ Yang-Mills connection is a connection whose curvature is harmonic with respect to the covariant derivative d_A.

When M has complex dimension 2, i.e., Kaehler surface, the Hodge $*$ operator determines a decomposition

$$\Lambda^2T^*M=\Lambda_+^2\oplus\Lambda_-^2$$

of the space of 2-forms, where $\Lambda_{\pm}^2$ denotes the eigenspace subbundle of $*$ of eigenvalue ± 1. Thus $*^2=id$ implies that the adjoint bundle $\mathfrak{g}_P=\boldsymbol{P}\times_{Ad}\mathfrak{g}$ valued 2-form $F_A=dA+(1/2)[A\wedge A]$ splits into $F^+=(1/2)(F_A+{*}F_A)$ and $F^-=(1/2)(F_A-{*}F_A)$, which are called the *self-dual* part and the *anti-self-dual* part of F_A respectively, where $\mathfrak{g}$ denotes the Lie algebra of G. Thus a connection A on a principal fibre bundle $\boldsymbol{P}$ over a Kaehler surface M being *Yang-Mills* is equivalent to $d_AF^+=0$ or $d_AF^-=0$.

But for a higher dimensional Kaehler manifold these formulae give us no meaning. Thus instead of using Hodge $*$ operator let us introduce another operator $\#$, which is defined in section 2 such as $\#=*^{-1}\circ L^{(n-2)}/(n-2)!$, where L means the multiplication by Φ. Then a connection A on a principal fibre bundle $\boldsymbol{P}$ over higher dimensional Kaehler manifold M being *Yang-Mills* is equivalent to $d_A\#F_A=0$ (cf. Proposition 3.1 (ii)).

Also let us define an operator $\tilde{\#}$ such that $\tilde{\#}$ is equal to $\#$ on $F^{2,0}+F^{0,2}+$

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