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NOTES ON M-SEMIGROUPS

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Introduction.

Let S be a torsion-free cancellative commutative (additive) semigroup $\supsetneqq\{0\}$. Let G be the quotient group of S. We assume $G\neq S$. For each subset A of G, we set $A^{-1}=\{x\in G: x+A\subset S\}$ and $(A^{-1})^{-1}=A^{v}$. If $A^{v}=A$ for an ideal A of S, then A is called a v-ideal of S. If S satisfies the ascending chain condition for v-ideals, then S is called a Mori-semigroup. If S is a Mori-semigroup and and if each ideal of S generated by two elements is a v-ideal, then S is called an M-semigroup ([**2**]). If each ideal of S is a v-ideal, then S is called a reflexive semigroup. The maximal number n such that there exists a chain $P_1\supsetneqq P_2\supsetneqq\cdots\supsetneqq P_n$ of prime ideals of S is denoted by $\dim S$. If $\dim S\geqq 1$, then S has a unique maximal ideal.

In this paper we study a semigroup version of a result ([**1**, Théorème 3]) of Querre. Our result is the following.

MAIN THEOREM. *Let S be a Mori-semigroup. Then the following conditions are equivalent:*

(1) $\dim S=1$ *and M^{-1} is generated by two elements for the maximal ideal M of S.*

(2) *S is a reflexive semigroup.*

(3) *Each ideal of S generated by two elements is a v-ideal.*

In [**4**] it is shown that the conditions (2) and (3) are equivalent and (2) implies (1). Therefore it is sufficient for us to show that (1) implies (2).

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1. Notations and Preliminaries.

Let $\boldsymbol{Z}$ be the set of integers and let $\boldsymbol{N}$ be the set of natural numbers. If for $v\in M$ and $u\in S$, there exists $n\in\boldsymbol{N}$ such that $nv\in(u)$, S is called a weakly

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