TSUKUBA J. MATH.
Vol. 14 No. 2 (1990), 489–496

# THE HOCHSCHILD COCYCLE CORRESPONDING TO A LONG EXACT SEQUENCE

Dedicated to Hiroyuki Tachikawa on his 60th birthday

By

Vlastimil DLAB and Claus Michael RINGEL

**1.** Let $k$ be a field, and $A$ an associative $k$-algebra with 1. Let $M, N$ be right $A$-modules. We denote by $H^{\cdot}$ the Hochschild cohomology of $A$. It is well-known that there is a natural isomorphism

$$\eta_{MN}:\mathrm{Ext}_A^t(M, N)\longrightarrow H^t(A, \mathrm{Hom}_k(M, N))$$

see Cartan-Eilenberg [CE], Corollary IX. 4.4. For $t\geqq 1$, the elements of $\mathrm{Ext}_A^t(M, N)$ may be considered as equivalence classes of long exact sequences, see Mac Lane [M], chapter III. Let

$$E=(0\longleftarrow M\overset{g_0}{\longleftarrow}Y_1\overset{g_1}{\longleftarrow}Y_2\longleftarrow\cdots\longleftarrow Y_t\overset{g_t}{\longleftarrow}N\longleftarrow 0)$$

be an exact sequence. We want to derive a recipe for obtaining a corresponding cocycle $A^{\otimes(t+2)}\rightarrow\mathrm{Hom}_k(M, N)$.

For $0\leqq i\leqq t+1$, let $Z_i$ be right $A$-modules, and for $0\leqq i\leqq t$, let $\beta_i: Z_i\rightarrow Z_{i+1}$ be $k$-linear maps. With $\beta=(\beta_0, \cdots, \beta_t)$ we associate a map

$$\Omega_{\beta}: A^{\otimes(t+2)}\longrightarrow\mathrm{Hom}_k(Z_0, Z_{t+1})$$

defined by

$$(a_0, \cdots, a_{t+1})\Omega_{\beta}=\bar{a}_0\beta_0\bar{a}_1\beta_1\cdots\bar{a}_t\beta_t\bar{a}_{t+1},$$

for $a_0, \cdots, a_{t+1}\in A$, where $\bar{a}_i$ denotes the scalar multiplication by $a_i$ (on $Z_i$); note that all maps will be written on the right of the argument, thus the composition of $\beta_0: Z_0\rightarrow Z_1$, and $\beta_1: Z_1\rightarrow Z_2$ is denoted by $\beta_0\beta_1$.

Given the exact sequence $E$ exhibited above, it clearly splits as a sequence of $k$-spaces, thus there are $k$-linear maps

$$M\overset{\gamma_0}{\longrightarrow}Y_1\overset{\gamma_1}{\longrightarrow}Y_2\longrightarrow\cdots\longrightarrow Y_t\overset{\gamma_t}{\longrightarrow}N$$

such that

$$\gamma_{i-1}\gamma_i=0,\quad g_{i-1}\gamma_{i-1}+\gamma_i g_i=1_{Y_i},\qquad \text{for } 1\leqq i\leqq t,$$

Received December 4, 1989.