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A CASE OF EXTENSIONS OF GROUP SCHEMES OVER A DISCRETE VALUATION RING

By

Tsutomu SEKIGUCHI*) and Noriyuki SUWA

Introduction.

Let $X\to Y$ be a cyclic covering of degree m of normal varieties over a field k. If m is prime to the characteristic of k and k contains all the m-th roots of unity, the Kummer theory asserts that the covering $X\to Y$ is given by a cartesian square:

$$\begin{array}{ccc} X & \dashrightarrow & \boldsymbol{G}_{m,k} \\ \downarrow & f & \downarrow \theta \\ Y & \dashrightarrow & \boldsymbol{G}_{m,k}, \end{array}$$

where θ is the m-th power map and f is a rational map of Y to the multiplicative group $\boldsymbol{G}_{m,k}$. On the other hand, if $m=p^n$ and $p=\text{char}.k>0$, the Witt-Artin-Schreier theory asserts that the covering $X\to Y$ is given by a cartesian square:

$$\begin{array}{ccc} X & \dashrightarrow & W_{n,k} \\ \downarrow & g & \downarrow \mathcal{P} \\ Y & \dashrightarrow & W_{n,k}, \end{array}$$

where $\mathcal{P}(x)=x^p-x$ and g is a rational map of Y to the Witt group $W_{n,k}$. Therefore, if one wishes to deform a cyclic covering $X\to Y$ of degree p^n over a field k of characteristic $p>0$ to a cyclic covering of degree p^n over a field of characteristic 0, it seems natural to consider the deformations of the Witt-Artin-Schereier exact sequence

$$0 \longrightarrow (\boldsymbol{Z}/p^n)_k \longrightarrow W_{n,k} \xrightarrow{\mathcal{P}} W_{n,k} \longrightarrow 0$$

over a field k of characteristic $p>0$ to an exact sequence of Kummer type

$$1 \longrightarrow \boldsymbol{\mu}_{p^n,K} \longrightarrow (\boldsymbol{G}_{m,K})^n \longrightarrow (\boldsymbol{G}_{m,K})^n \longrightarrow 1$$

over a field K of characteristic 0. From this point of view, it seems most

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