TSUKUBA J. MATH.
Vol. 14 No. 2)1999), 437—457

CONSTRUCTION OF INVARIANTS

By

Akihiko GYOJA

1. Introduction.

Let G be a connected reductive group defined over the complex number field $\boldsymbol{C}$, V a finite dimensional vector space and $\rho: G\rightarrow GL(V)$ a rational representation of G. Such a triplet (G, ρ, V) is called a *prehomogeneous vector space* if V has an open G-orbit, and called *irreducible* if ρ is an irreducible representation. A complete list of irreducible prehomogeneous vector spaces is given by M. Sato and T. Kimura [12]. The purpose of this paper is to construct explicitly an irreducible relative invariant for every irreducible prehomogeneous vector space. If (G, ρ, V) and (G', ρ', V') are in the same castling class, then an irreducible relative invariant of (G, ρ, V) can be constructed from that of (G', ρ', V'). (See proposition 18 in [12, section 4].) Hence it is enough to consider irreducible reduced prehomogeneous vector spaces. (See [12, section 2] for the generalities concerning the castling transformations.) In the tables I and II of [12, section 7], irreducible relative invariants are given except for the following six cases;

(6) $(GL(7), \Lambda_3, V(35))$,

(7) $(GL(8), \Lambda_3, V(56))$,

(10) $(SL(5)\times GL(3), \Lambda_2\otimes\Lambda_1, V(10)\otimes V(3))$,

(20) $(\mathrm{Spin}\,(10)\times GL(2), (\text{half spin})\otimes\Lambda_1, V(16)\otimes V(2))$,

(21) $(\mathrm{Spin}\,(10)\times GL(3), (\text{half spin})\otimes\Lambda_1, V(16)\otimes V(3))$,

(24) $(GL(1)\times \mathrm{Spin}\,(14), (\text{half spin}), V(64))$.

Irreducible relative invariants of (6) and (7) are constructed by T. Kimura [8], and that of (20) is constructed by H. Kawahara [7]. (Concerning a construction of an invariant of (7), see the last section of the present paper.) Hence our task is to construct irreducible relative invariants of (10), (21) and (24).

Received July 5, 1989. Revised December 1, 1989.